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Performance Analysis and Attribution with Alternative Investments[1]

This Version: February 12, 2022

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Executive Summary

- This white paper provides a summary of existing methods and current research on performance analysis and attribution for complex portfolios that include investments such as hedge funds and private investment funds. We summarize the main challenges of analyzing portfolios with illiquid assets and provide updated analysis for several important asset classes.
- Using hedge fund return data from a variety of sources, we estimate excess returns using a factor-model approach. Results indicate substantial variation in performance and risk exposure estimates across different data providers. However, most strategies exhibit statistically significant exposure to global stocks, a small stock factor, and an illiquidity factor. We document that risk-adjusted performance (alpha) for hedge funds has averaged roughly 3-5% over the period from 2004-2021 though performance has declined on average since the Global Financial Crisis.
- Updated performance estimates for private equity funds suggest continued superior performance relative to public market benchmarks matched on portfolio industry and geography characteristics. The performance of private credit funds relative to a public benchmark has been more mixed in recent years.
- We present a method for deal-level performance attribution of buyout investments. Results include a novel analysis which controls for industry trends in EBITDA multiples and leverage. We find evidence of positive GP-related performance. Much of GP-related contribution is related to higher leverage, but the fraction of the leverage contribution has been declining in recent years.
- Real estate and other real assets display a trend toward weaker market-adjusted performance in recent years. We also discuss the challenges associated with identifying appropriate benchmarks given data limitations and the heterogeneity of real assets.

1. Introduction

As private investment opportunities continue to expand, investors have an increasingly complex array of assets to consider.[2] Many institutions are well-suited to benefit from investments in illiquid securities, because they have long time horizons and fewer constraints (Gilbert and Hrdlicka, 2015). This has resulted in a sustained shift toward institutional portfolios holding larger shares of assets in alternative investments.[3] The effects of this shift have been

[2] For a recent review of private market trends see McKinsey Global Private Markets Review (2021).
[3] See, among others, see Hochberg and Rauh (2013) and Binfare, Brown, Harris, and Lundblad (2019).

examined by empirical studies. For example, Lerner, Schoar, and Wang (2008) find that the shift by endowments toward higher allocations to alternative investments generates higher returns as well as a potential return benefit related to skill. More recently, Binfarè et. al. (2019) find that many endowments, especially larger ones, generate excess returns by tilting allocations toward better performing alternative assets and reliably selecting above average managers (especially in venture capital). Alternatively, findings by Cavagnaro, Sensoy, Wang, and Weisbach (2019) suggest that investment acumen is important, but that not all institutional investors earn systematically better returns. While these institutional portfolios are better positioned to invest in illiquid assets, there are also concerns that private investment opportunities may be getting crowded and expected gains may be declining especially in private equity and hedge funds.

1.1 Goals and objectives of performance analysis and attribution

Regardless of the potential opportunities, the growth of alternative assets has generated practical challenges for portfolio management. In particular, the proliferation of illiquid assets in private fund structures across a global asset marketplace has made basic issues like asset allocation, risk measurement and performance attribution increasingly difficult. Consequently, evaluating the potential benefits of alternatives, even ex post, is not straightforward.

A central assumption in modern portfolio theory is the ability of investors to trade in all assets. The literature often references a benchmark "market portfolio" which includes all available assets. Yet, portfolios increasingly invest beyond publicly-traded stocks and bonds into assets with no obvious public alternative. In addition, many investors cannot invest in the full spectrum of investments because of regulatory constraints, operational limitations, or limited access. In practice, investors often define a strategic allocation across the asset classes in which they invest. Increasingly these custom benchmarks include private funds, and in many cases even basic characteristics of the underlying assets may not be well-known (such as historical standard deviations and correlations with other assets) because it is hard to observe true periodic returns. In this sense, there is an increasingly vague notion of what actually constitutes the investible universe and thus the appropriate "market portfolio." In turn, this ambiguity makes the process of performance analysis and attribution difficult as compared to examination of liquid portfolios with all publicly-traded assets.

As a case in point, consider a private equity fund that earns an annualized internal rate of return (IRR) of 10% over a given evaluation period while a diversified (passive) index of publicly-traded stocks earns an annualized return of 8%. As part of the due diligence process for investment in the manager's next fund, the investor seeks to understand if the PE manager is skilled. A naïve assessment of the fund suggests it is superior to the public benchmark. However, there are two immediate concerns that the investor must consider. First, the fund's strategy may be systematically riskier than the public benchmark, so the fund may not have outperformed on a risk-adjusted basis. Second, the relative performance could be the result of luck, and thus is not a reliable measure of manager skill. In the case of a fund that holds only liquid publicly-traded assets, a simple regression analysis of periodic returns can be used to determine risk-adjusted performance (i.e., alpha) and its statistical significance. However, for a private fund, there is typically no reliable market value information and so a basic regression analysis is infeasible. But beyond this, there are other important considerations such as the timing of fund cash flows (contributions and distributions) and the potential for important differences in the types of assets the fund holds relative to the public index that make the benchmark inappropriate (e.g., geography, sector and size).

This simple example illustrates the key aspects of performance analysis and attribution we seek to examine in this white paper: in an increasingly complex investment universe how can one identify *statistically reliable risk-adjusted performance* for illiquid assets and portfolios? While this is a simple concept, it is a practical challenge that requires defining appropriate benchmarks and analysis methods that incorporate accurate measures of return (e.g., unsmoothed) and more complex measures of risk (e.g., illiquidity). At a broad portfolio level, the most common evaluations are made relative to a strategic benchmark portfolio, but even these seemingly simple comparisons can have serious problems. First, the comparison benchmark must be appropriate. That is, it should demonstrate the attributes that correspond to the strategies and goals of an investment. Additionally, for a proper apples-to-apples comparison a benchmark should be investible and transparent. This comparison is crucial to conveying the right information around portfolios and funds which is further complicated by issues like illiquidity and access. For example, some hedge fund indices have been criticized because they do not provide a transparent view of the funds in the index. Likewise, venture capital benchmarks have been criticized because many investors lack access to top-performing funds.

Illiquid assets have other unique features that create complexity when measuring performance. For example, the defining characteristics of illiquid assets is that investors cannot trade in and out of positions quickly and cheaply. As such, some investors in illiquid assets might perceive that they are insulated from rapid price fluctuations that can plague liquid markets during times of stress. However, this perceived benefit is only of economic value to the extent that it reduces the opportunity for investors to liquidate assets in a fire-sale situation because of panic (or some other sub-optimal reason). On the other hand, as illiquid assets are more difficult to trade, investors who prefer to (or must) stay close to a certain asset allocation may face the risk of needing to take steep discounts in order to rebalance such as those documented by Nadauld, Sensoy, Vorkink, and Weisbach (2019) in the secondary market for Private Equity (PE) funds. Additionally, illiquid assets take more effort in the form of due diligence and management leading to firms charging investors higher fees. As found by Harris, Jenkinson, Kaplan, and Stucke (2018) this need for investors to use intermediaries to gain access to illiquid assets, may be suboptimal.

1.2 Global Investment Performance Standards (GIPS)

When considering the general framework of performance analysis, the investment management industry has made great strides in defining comparable methods and measures. One example of such efforts is the creation of the Global Investment Performance Standards (GIPS) by the CFA Institute. The purpose of GIPS is to create a standard which will give firms and investors trust around the calculation of performance metrics.[4] Investors gain the ability to compare returns on a standardized basis which should provide confidence in the ability to select quality asset managers. Following GIPS also provides standards for performance metrics that are reported to oversight boards. Recent updates have attempted to make GIPS more applicable to a broader set of assets, but these attempts fall short of providing useful tools for properly calculating accurate risk-adjusted returns for illiquid assets.[5] More specifically, GIPS standards provide a standardized method for calculating and reporting performance measures, but they do not address the challenges of creating economically meaningful comparisons to liquid assets for risk and return metrics (e.g., standardized methods for unsmoothing returns or high-frequency pricing).

[4] See, https://www.gipsstandards.org/wp-content/uploads/2021/03/2020_gips_standards_firms.pdf.
[5] See, http://www.gipsstandards.org/wp-content/uploads/2021/02/2020_gips_standards_asset_owners.pdf.

1.3 Data and return transparency

To properly conduct performance analysis and attribution, one must carefully consider the availability of data and the transparency of the returns. Given the nature of private illiquid assets, data availability has been an ongoing challenge as these assets and funds tend to be operated as legal entities that are not required to disclose more than a minimum amount of information publicly (e.g., basic characteristics in the U.S. SEC filings of Form ADV). In other cases, direct holdings of assets (such as real estate and equity co-investments) may not have any required reporting. While data are becoming more available, the lack of data still presents a significant barrier to analyzing performance for many investors. Additionally, while these data may become more available, the lags in reporting and low frequency of observation pose additional challenges. For example, quarterly fund net asset values (NAVs) are not true market values and are only available with a substantial lag. While efforts have been made to create more frequent valuation measures, such as Boyer et al. (2018) and Brown, Ghysels and Gredil (2020) in PE, the majority of usable data are reported on a monthly, quarterly, or even annual basis. Additionally, this creates limitations for model estimation and inference. In addition, the purchase and sale of individual private assets (e.g., real estate properties and portfolio companies) only occur when a willing buyer and seller in a highly illiquid market can negotiate a transaction price agreeable to both. The effect of this two-sided search on pricing patterns is demonstrated in Sagi (2021) which finds that the value of commercial real estate does not follow the same random price process commonly observed in public markets.[6] With these illiquid assets, there are bespoke items essential to the market making which creates a complicated timing mechanism on pricing.

The remainder of the paper proceeds as follows. Section 2 provides an overview of the prevailing methods of performance analysis and how these apply to illiquid assets. Section 3 considers specific applications for hedge funds, private equity and credit, and real estate and other real assets. Section 4 provides an overview of broader portfolio analysis and attribution. Section 5 concludes.

2. Performance Analysis and Attribution Methods

[6] We discuss this further below, but the most common model of a publicly-traded asset price process is a random walk with a "drift" that represents the expected return of the asset. Sagi shows both theoretically and empirically, that illiquid assets do not follow a random-walk with drift.

2.1 Traditional framework and estimation issues

To begin our overview of current methods of performance analysis and attribution, we first consider the traditional framework for portfolio analysis. In its most basic form, modern portfolio theory rests on the finding that in equilibrium there will be a market portfolio of all available assets (that is optimal and efficiently priced) and sits on the efficient risk-return frontier. Any investor acting optimally will hold a portfolio similar to the market portfolio and lever it up or down with risk-free borrowing or lending based on their individual risk preferences. In addition, rational investors will diversify away as much idiosyncratic risk as possible. In a market with private assets not available to all investors, some investors will hold a portfolio comprised of private assets and their risk-return frontier may differ from those investors without access to private assets. If investors in private assets are scarce, or require an illiquidity premium, the returns available to private asset investors can be greater than would be suggested by their fundamental risk profile. In theory, this may leave investors who can only access publicly-traded assets with a suboptimal portfolio. Thought of differently, there will be a group of investors who cannot fully access all assets and reach an extended efficient frontier. To the extent that private assets provide diversification benefits, public-only investors may also be under-diversified. This potential discrepancy has been part of the motivation for allowing retail investors to access to private funds.[7]

On the other hand, private asset investors do not invest in the entire "market" for private assets, because it is impractical. For example, even the most active private investors make only a limited number of commitments to private funds—typically less than 20 per year. This means that some private fund investors could be taking on significant idiosyncratic risk in their portfolios—especially if they focus private fund investments into only certain strategies or geographies. This additional idiosyncratic risk must be understood by investment managers (and fiduciaries) to properly understand the overall risk-return benefit to being invested in private assets (see, for example, Gredil, Liu, and Sensoy, 2020). In this setting, a great importance is laid on the investment decisions of portfolio managers, and consequently the evaluation of such decision making using performance metrics. In addition, since portfolio managers are typically

[7] See recent recommendations by the SEC's Asset Management Advisory Committee (https://www.sec.gov/files/final-recommendations-and-report-private-investments-subcommittee-092721.pdf) and Brown et al. (2020).

being rewarded for such exposure to idiosyncratic risks, it is important to understand the attribution of risk-adjusted performance for illiquid assets to mitigate agency costs related to undesired speculation (or conservatism).

Another element that adds complexity is illiquidity. Investors experience risks due to the illiquidity of an asset when they are unable to buy or sell the full position in the timing they desire or for the price considered to be fundamental value. The existence of an illiquidity premium in traditional assets like stocks and bonds has been well documented.[8] Such a premium for illiquidity is also observed for more illiquid assets such as private equity and hedge funds.[9] Notably, while the premium for exposure to illiquidity risk has been shown to exist, a single preferred measure for how to classify this risk and related premium is still a matter of debate in the literature. Ang, Papanikolaou, and Westerfield (2014) propose that the uncertainty around the interval of time where an asset is untradeable, and an investor is forced to maintain an illiquid position, is the driving factor for the required premium.[10] Jansen and Werker (2021) examine the shadow costs of illiquidity such as the loss of wealth (consumption) derived from the inability of investors to have an optimal asset allocation.[11]

As illustrated in the PE fund example above, characterizing performance by return alone is unlikely to be appropriate. Instead, methods that allow for more careful analysis of the specific risks associated with illiquid investments, whether they be market-wide or unique to illiquid assets, are a must.

2.2 Delegated portfolio management and fees

Before turning to specific performance evaluation methods, it is useful to consider more broadly the options available to an asset owner when deciding how their portfolio is managed. There are a number of different methods of investing, each of which come with benefits and costs. At one end of the spectrum is an entirely passive portfolio of publicly-traded assets. A fully-diversified passive portfolio can be obtained easily and at very low cost. For example, portfolios of index funds provided by large asset managers like Vanguard and Blackrock can be

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[8] See, for example, Pastor and Stambaugh (2003) , Acharya and Pedersen (2005) , and Driessen and De Jong (2012).

[9] See, for example, Franzoni, Nowak, and Phalippou (2012) and Sadka (2010).

[10] Ang, A., Papanikolaou, D., and Westerfield, M. (2014). Portfolio choice with illiquid assets. Management Science, 60(11):2737–2761.

[11] Jansen, Kirsty A.E. and Bas J. M. Werker. (2021) The shadow costs of illiquidity. The Journal of Financial and Quantitative Analysis, forthcoming

utilized for annual fess of less than 0.10%. However, as soon as an investor chooses to access private assets there is a more complicated delegated portfolio management problem. Investors must either build in-house expertise to evaluate opportunities or rely on external advisors. Both can be quite costly. In addition, a variety of investment structures exist ranging from direct investments, co-investments with sponsors, separately managed accounts, comingled fund investments, and fund-of-fund investments.

Direct investment into private assets let investors avoid some external asset management fees. However, such direct investment may come with a large investment amount beyond the scope of many investors. Additionally, the burdens of due diligence and monitoring associated with a direct investment fall solely on the investor. Without significant economies of scale these costs can exceed external management fees. Investors face a similar, but less extreme, situation for co-investments where an investor puts in capital alongside a sponsor. For co-investments, there are typically benefits from shared diligence, lower fees, and smaller check size, however, the opportunities for co-investments are limited for the majority of investors. Outside managers will often provide the opportunity for a separately managed account (SMA) which will provide many of the benefits of delegated asset management but with high degrees of customization and lower fees. Depending on the types of assets under management, the minimum commitments for an SMA can range from roughly $10-100 million USD making SMAs a practical alternative only for large institutional investors and ultra-high-net-worth individuals.

The lack of access to direct investments and co-investments (managed in-house or via an SMA) creates a need for comingled funds. These funds pool the capital of their investors and then invest in assets for fees that typically include both a percentage of assets under management as well as incentive fees based on performance. Two clear benefits to investing in funds are i) access to a broader (and more diversified) set of assets and investment strategies and ii) the potential to select managers with superior skill that can provide high risk-adjusted returns (i.e., alpha). Two costs to investing in funds are i) higher external management fees and ii) generally less control over the timing and type of investment decisions especially in the case of closed-end investment vehicles. For example, private fund investors typically delegate investment decisions (i.e., invest in a "blind pool") and have little control over the timing of investments and exits.

In some situations, commitments to primary funds can present substantial challenges. For example, desired investments into niche markets or limited access to certain managers can make primary fund commitments unwieldy or even impossible. In these cases, funds-of-funds can provide investors better access to certain assets and investment strategies. However, funds-of funds entail incurring an extra layer of fees. The empirical evidence on alternative assets funds-of-funds is mixed. Some studies find that the additional layer of fees is offset by better selection of underlying funds.[12] However, Andonov (2020) finds that, in general, investors would receive higher returns by investing passively in the public market rather than alternative assets. Notably, Andovov also finds larger investors who invest more in primary funds enjoy higher returns. In this case, the ability of investors to gain access to the desired investments in illiquid assets again poses a major driver to understanding the attributions of performance.

For the purposes of this analysis, it is important to keep in mind that determination of the optimal mechanism for investing, whether it be through direct investments or fund-of-funds requires an ability to precisely understand risk-adjusted performance net of all investment costs and attribute it to skill (or a lack thereof). Furthermore, the underlying issues of employing external portfolio managers should considered in the context of principal agent theory.[13] Any time there is separation between the owner of the capital and those who manage it, the contractual arrangements, return and risk assessment, and fees associated with that relationship can have a major impact on portfolio management decisions and overall performance.

2.3 Benchmark selection

At a basic level there are two methods for evaluating the returns: absolute performance and relative performance. Absolute performance conveys the change in total value during a specified time period. For example, the stock of Apple (AAPL) gained 7.5% during a year. Performance measured in absolute terms provides insight into how an investment is performing in terms of the wealth it generates or destroys. This is useful information for investors who need to use assessments of actual value to make real economic decisions. Investors ultimately care about what their investments will allow them to consume, and as a consequence derive economic benefit from the inflation-adjusted or "real" returns of a portfolio. This often motivates performance benchmarks like the Consumer Price Index plus a "real return" of 5% (CPI+5%) as a long-term investment objective. In our example, if inflation in US dollars totaled 4% during the

[12] See for example Harris, Jenkinson, Kaplan, and Stucke (2018).
[13] For example, as discussed in such as in Bhattacharya and Pfleiderer (1985).

investment period, then the real return on AAPL would be just 3.5% (7.5% - 4%) which is below the targeted real return.

While comparison to an inflation-adjusted benchmark is a measure of relative performance, it does not take into account the riskiness of the investment. Relative performance is best measured as the difference between the absolute performance and the performance of an appropriate benchmark. In the case of AAPL, the performance relative to a benchmark like the NASDAQ Composite will be negative when the benchmark has higher returns. However, performance inference from this simple comparison assumes that the systematic risk of AAPL is the same as for the Nasdaq composite (e.g., AAPL has a market beta of 1.0) and that the Nasdaq composite is in fact the correct benchmark.

In practice, an appropriate benchmark should be chosen based on the strategy or goal of an investment. The notion of appropriate benchmarking is not a new one. Grinblatt and Titman (1989) discusses the importance of identifying the correct benchmark in portfolio performance evaluation. As anyone who has purchased a home knows, the set of comparables chosen can have a significant impact on the appraised valuation of a home. The same holds when considering a portfolio of assets. Continuing the simple single stock example from above, comparing AAPL to the Nasdaq Composite may show underperformance yet comparison to another equity index might show outperformance.

In practice, there are two schools of thought on selecting the right benchmark. The first is to pick a benchmark that matches the underlying portfolio assets as closely as possible. This approach focuses on making an apples-to-apples comparison with the view that investing in the benchmark was an alternative to investing in the fund. For example, a small-cap value index might be used for buyout funds and a REIT index might be used for real estate private equity funds. These benchmarks might be adjusted for the relative riskiness of the investment being evaluated (more on this below). This is a preferred approach when evaluating the skill of a manager. The second approach entails picking a benchmark that characterizes the broad asset class exposure. This approach considers the performance of the fund as part of a broader portfolio and assumes that diversifiable risks (e.g., from sector or size) do not matter. For example, a total stock market index might be used for buyout funds. This approach is preferred when evaluating how a fund contributes to overall portfolio performance (and of course, can also be adjusted for the level of an investments systematic risk).[14] At a more granular level, factor-based benchmarking can provide additional insight into performance and we cover this in detail below as well.

2.4 Portfolio performance measurement basics

A standard practice in portfolio management is to look beyond the simple return, whether absolute or relative, and consider performance scaled by risk. The most common scaled performance metric is the Sharpe Ratio, which measures the annualized return of a portfolio (r_P) above the risk-free rate (r_f) scaled by the annualized standard deviation of those excess returns (σ_P).

$$Sharpe\ Ratio = \frac{r_P - r_f}{\sigma_P}.$$

This ratio provides additional context to portfolio performance by assessing the tradeoff between the returns and risk as measured by portfolio volatility (thus a higher Sharpe ratio is preferred).

Another commonly used metric is the Information Ratio which is the difference between the portfolios return and a benchmark return (r_B) divided by the annualized standard deviation of the tracking error of the portfolio return (σ_E).

$$Information\ Ratio = \frac{r_P - r_B}{\sigma_E}.$$

In this case, we have the *relative* return of the portfolio scaled by the volatility of those returns to give a value for the average excess return per unit of risk versus a benchmark. Again, we can see how the choice of benchmark is important. As we discuss below, the excess return in the information ratio can be calculated using a multi-factor risk model instead of a single benchmark.

Many investors have made the observation that financial risk is fundamentally about negative returns and so risk metrics like standard deviation that penalize “upside” risk are potentially less appropriate than measures that only consider downside risk. Like the Sharpe Ratio, the Sortino Ratio examines the return of a portfolio relative to the risk-free rate, but instead scales it by downside risk as measured by the lower semi-deviation (i.e., variation in returns below the mean excess return, σ_D).

[14] These two schools of thought are also important for investors who operate with risk budgets. The first school minimizes tracking error and more precisely evaluates manager-specific attributes while enhancing the allocation effects at the aggregated portfolio level. The second school of thought does the reverse.

$$Sortino\ Ratio = \frac{r_P - r_f}{\sigma_D}.$$

There are a number of challenges with these types of performance measurements when portfolios hold illiquid assets. As pointed out by Korteweg (2019), a challenge in the PE space is that there is no consensus on the best empirical approach to valuing risk and return, let alone the correct method for constructing appropriate benchmarks. As illiquid assets values are observed infrequently and with substantial error, the methods of comparison or the ability to create accurate return series become arduous. Gompers, Kaplan, and Mukharlyamov (2016) find that in practice, fund managers and investors tend to rely on measures that are not adjusted for risk and do not utilize periodic returns such as internal rate of return (IRR) and cash multiples. Subsequently, we discuss illiquid asset performance metrics, but the difficulties surrounding the evaluation of the risk and return of illiquid assets creates an even greater challenge for benchmarking.

2.5 Risk modeling: The factor approach to separating alpha from beta

The primary goal of portfolio performance analysis and attribution is to evaluate how the decisions made at the time of investment impacted the returns of the portfolio. The framework for the analysis ties to portfolio theory and the one-factor Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965). In the simple CAPM context, performance analysis can be used to determine if investment decisions resulted in a better outcome than holding an appropriately levered benchmark market portfolio. As noted already, when using a benchmark portfolio that includes only publicly-traded assets to evaluate a portfolio that includes private assets, superior relative performance can come from both investment decisions as well as diversification benefits from access to a wider set of investments (i.e., potentially even without superior portfolio management skills, see Goetzmann, Gourier, and Phalippou, 2019).

In recent years, the factor model approach has gained popularity as a method to understand and characterize portfolio returns. In essence, the factor model approach commonly used today is derived from insights originally developed by Richard Roll in his Arbitrage Pricing Theory and extended through the development of empirical models by Fama and French (1993) along with many others since. The basic insight derives from the observations that asset returns (especially equities) have risk characteristics that are not captured by the single-factor market model (i.e., CAPM). For example, Fama and French (1993) utilize the, now ubiquitous, size factor (so-called SMB or small-minus-big) and value factor (so-called HML or high-minus-low book-to-market ratio) as well as bond market factors related to maturity and default risk. Subsequent research has proposed many additional risk factors.

As discussed by Ang (2014), current factor theory states that assets earn premiums because they have exposure to two types of underlying risk factors: fundamental-based factors and investment-style factors. For example, in the case of the CAPM beta, assets that are highly exposed to broad market movements demand higher returns to compensate investors for bearing that risk. Similar relations hold for other factors for which investors expect to earn risk premia (on average). Consequently, one way to write the expected return, $E[R_t]$, of an asset for period t is to assume it is a linear function of N different risk factors, X_i, with unique exposures, β_i, to each factor so that

$$E[R_t] = r_f + \beta_1 * E[X_1] + \beta_2 * E[X_2] + \ldots + \beta_N * E[X_N] .$$

In this case, a modeler would need to know the N different risk factors and their expected returns. Unfortunately, current practice does not provide clear guidance on a specific set of risk factors (or even the number of factors) or an easy way to estimate expected returns. However, factor models are still extremely useful for examining historical risk-adjusted performance as well as considering overall risk exposure on an ongoing basis. There are a variety of commonly used factors and estimating a portfolio's historical risk-factor exposure and risk-adjusted performance is as simple as estimating an ordinary least squares (OLS) regression. For example, the risk-adjusted performance, α, of an equity portfolio evaluated using the Fama and French 3-factor model is easily obtained by estimating the coefficients of the model

$$R_t - r_f = \alpha + \beta_1 * (R_{M,t} - r_f) + \beta_2 * SMB_{,t} + \beta_3 * HML_{,t} + \varepsilon_t$$

where R_t is the periodic return of the portfolio of interest, $R_{M,t}$ is the periodic market return, r_f is the risk-free rate, and ε_t is the residual return unexplained by the model. If the estimated value of α is statistically greater than zero, the portfolio provided superior risk-adjusted performance relative to a portfolio with similar exposure to the three factors under consideration. Moreover, it is simple to calculate a multi-factor information ratio by annualizing the estimate of α and dividing by the annualized standard deviation of ε_t.[15] In the next session, we apply a factor-model approach to the performance analysis of hedge funds.

Of course, estimating factor exposures with a regression model is not directly feasible for private assets that lack periodic return data. However, it is still feasible to infer factor exposures indirectly. One approach is to identify comparable public assets (e.g., match on size, industry, and geography) and assume that private assets have similar exposure. A substantially more complicated, but more direct, approach is to use a statistical model to infer an approximated return series and use this in the factor model estimation. Brown, Ghysels, and Gredil (2021) provide one such method for venture capital, private equity buyout, and real estate funds.

Another challenge to the factor-model approach is the need to decide on a set of factors. Specification of the set of factors has become increasingly complicated with the massive increase in identification of factors that appear to earn risk premia. For example, Harvey and Liu (2019) have created a database of over 400 factors published in top finance journals and discuss how many of these are found significant only as a consequence of luck. Despite this large "factor zoo", it is still possible to specify a parsimonious factor model based on an intuitive set of factors. For example, a simple factor model might use just three broad capital market factors: global stocks, global bonds, and commodity (or real asset) returns.

2.6 Attribution analysis

The goal of performance analysis and attribution goes beyond wanting to determine total risk-adjusted performance. Portfolio managers often tilt investments based on views related to industry sector, geography, manager ability, asset-specific expectations, and tactical timing considerations. Consequently, understanding the attribution of value creation (or destruction) based on deliberate tilts is important for evaluating the skill of portfolio managers. For each of the choices, the main goal is to question whether the investment criteria were appropriate when compared to the broader market or a different selection of assets. For example, a portfolio manager can evaluate the historical allocation to sectors and consider whether a different mix of sectors would have yielded higher or lower returns. The same can be done for any number of investment criteria and while the outcomes may not be predictive, this analysis provides a deeper

[15] Typically an annualized alpha, α^*, is calculated by compounding the estimated alpha so that $\alpha^* = (1+ \alpha^*)^T - 1$ where T is the number of periods per year in the dataset used for estimation (e.g., 12 for monthly data). The standard deviation of ε_t is annualized my multiplying the estimated standard deviation by the square root of T.

understanding of what determined a portfolio's historical returns. There are well-established methods for attribution in portfolios with only publicly-traded assets, so in our analysis we focus on portfolios with illiquid assets.[16] For example, Brown, Ethridge, Johnson, and Keck (2021) provide a method for private fund performance attribution that examines portfolio management decisions relative to an investible universe of private funds. We discuss this model in more detail in Section 4.

Attribution analysis can also be conducted at the individual investment level. For example, decomposing performance drivers is common in the due diligence process for private equity buyout funds. Investors seek to understand how top line revenue growth, operating efficiency gains, multiple expansion and financing policy contribute to returns for individual deals. Performance can often be further decomposed into changes related to industry- or market-wide changes versus deal-specific changes. For example, in a buyout deal it is common to attribute the expansion of the EBITDA multiple between entry and exit to an industry-wide change and a deal-specific change. For a generalist fund, the implication might be that an industry-wide change could reveal the ability of the manager to pick industries and a deal-specific change could inform the ability of the manager to make the earnings of a specific company more highly valued. In Section 3 we provide a detailed description of a specific deal-level attribution analysis and examine a large dataset of global buyout transactions.

2.7 ESG performance and factors

Environmental, social, and governance (ESG) issues have become increasingly important to investors in recent years. While a detailed analysis of ESG metrics and related performance is beyond the scope of this paper, no analysis would be complete without some discussion. Current thinking about the potential benefits of ESG is divided into two (not mutually exclusive) camps. First, it is argued that firms with better ESG policies operate more efficiently and this, in turn, produces superior performance. Friede, Busch, and Bassen (2015) conduct a meta-study of more than 2,000 empirical papers and document that a large majority of studies find a positive relation between ESG and corporate financial performance. Second, it is argued that positive ESG attributes help mitigate down-side risk. For example, a recent analysis by Albuquerque, Koskinen, Yang, and Zhang (2020) find firms with better environmental and social activities (ES

[16] See, for example, Bacon (2019).

ratings) experienced better operating and stick return performance during the 2020:Q1 period of severe market dislocation related to the COVID-19 pandemic.

The documented superior investment performance of companies with high ESG ratings has been termed the "greenium." However, a sustained return greenium is at odds with equilibrium theory. If information about ESG characteristics is widely available and some investors prefer investments with better ESG ratings, then there should exist a *valuation* premium (and thus a return penalty) for these companies. Simply put, investment assets related to companies with high ESG ratings should be more expensive, and those more expensive assets should subsequently provide lower investment performance, all else the same. This suggests that the documented out-performance of assets with high ESG ratings is likely to be a one-time shift to higher valuations as investors take notice of ESG and adjust their portfolio holdings—and not a long-run sustainable strategy for earning superior risk-adjusted returns. Recent theory and evidence by Pastor, Stambaugh, and Taylor (2020, 2021) supports this view.

Nonetheless, investors increasingly need to know the ESG-related characteristics of their portfolios. While many providers now generate ESG ratings for public companies, evaluating private companies is more difficult. Private companies are less likely to have regulatory reporting requirements or pressure from vocal public shareholders to generate ESG-relevant information. Likewise, most private companies are small and do not have resources devoted to ESG data collection and reporting. This gap in data provides operational challenges to comprehensive ESG reporting for portfolios with substantial private asset allocations. One method for inferring ESG exposure is to utilize an ESG factor-mimicking portfolio and include it in a factor model as described above in Section 2.5. For example, a factor can be created by calculating returns form a portfolio that is long companies with high ESG ratings and short companies with low ESG ratings.[17] Portfolio managers can interpret the estimated coefficient on such a factor as the exposure their portfolio has to ESG. For private assets where it is not possible to estimate ESG factor exposures, portfolio managers can use the exposures for comparable assets (e.g., by matching on similar industry, size, and geography).

[17] See, for example, Pastor, Stambaugh, and Taylor (2021)

2.7 Illiquid asset performance metrics

Given the lack of observable periodic returns, other metrics are commonly used to evaluate the performance of illiquid assets. One of the most common metrics is the valuation multiple which simply measures the ratio of the exit (or current) value of an asset to the value initially invested. For deal-level analysis, this is commonly referred to as the multiple on invested capital (MOIC) and at the fund level, this is commonly referred to as the total value to paid-in-capital (TVPI) ratio. These multiples represent an intuitive measure of performance where a multiple greater than 1.0 is considered a profitable investment. While these multiples are easy to calculate and intuitive, there is no adjustment for risk nor any adjustment for the length of the investment period.

Another common performance metric is the internal rate of return (IRR) which measures the annualized return of an investment implied by the investment cash flows (CF_t). The IRR is defined as the discount rate that sets the net present value (NPV) of all periodic cash flows to zero, or more precisely

$$0 = \sum_{t=0}^{T} \frac{CF_t}{(1 + IRR)^t}$$

where the first investment is made at time t=0 and the last cash flow occurs at terminal date T. In practice, the terminal date cash flow is often assumed to be the current net asset value (NAV) of the investment. The IRR is often compared to market rates of return over the same time frame or alternatively, an appropriate opportunity cost of capital, to determine if an investment was good or bad. Because of peculiarities related to the timing of cash flows, IRR comparisons can be misleading. Nonetheless, the IRR is easy to calculate and intuitive. However, there is no explicit adjustment for risk and it assumes that cash flows are reinvested at the IRR which can also prove misleading if the IRR is very high or very low.

An alternative metric is the public market equivalent (PME) which, like the multiples method, measures the ratio of cash inflows to cash outflows. However, the cash flows are future values calculated using realized rates of return for a public market benchmark. There are different flavors of PME, but the method of Kaplan and Schoar (2005) is generally considered the best and used most in research. Specifically, the KS-PME is

$$PME = \frac{\sum_{t=0}^{T} CF_t^{inflow}(1 + R_t^M)}{\sum_{t=0}^{T} CF_t^{outflow}(1 + R_t^M)}$$

where R_t^M is total return on the public market benchmark between time *t* and terminal time *T*. A PME greater than 1.0 means the investment returned more than the public benchmark and vice versa. A key advantage of the PME is that it provides for explicit comparison to a public market benchmark and provides a precise estimate for the total relative performance. For example, a PME of 1.20 means that the investment outperformed similarly-timed investments in the benchmark by a total of 20% over the investment horizon.

One drawback of the PME method is that it does not adjust for the investment time horizon. Clearly a PME of 1.2 earned over 5 years is much preferred to a PME of 1.2 earned over 10 years. The Direct Alpha (DA) method of Gredil, Griffiths, and Stucke (2014) effectively converts PMEs into annualized excess returns. More specifically, DA measures the annualized excess return over the benchmark return by calculating the IRR of the future value of all cash flows obtained (as with PME) using returns on a public market benchmark so that

$$0 = \sum_{t=0}^{T} \frac{CF_t(1+R_t^M)}{(1+DA)^t} \ .$$

A DA greater than 0% (less than 0%) means the investment returned more (less) on average than the public benchmark. It is important to note that for both PME and DA calculations the benchmark can have a significant impact on the outcome and interpretation. Consequently, the selection of the proper benchmark is important and we discuss this in detail in the next section.

Another drawback of the PME method is the assumption of a beta of 1.0 relative to the benchmark. Korteweg (2019) surveys the empirical literature examining risk estimates of private equity funds and shows that most estimates result in betas greater than 1.0. One solution to this problem is to use a levered benchmark return in PME and DA calculations. For example, assuming a beta of 1.3 and adjusting benchmark returns using the standard market model. A more elegant solution is proposed by Korteweg and Nagel (2016) which develops the *generalized* PME (GPME) method based on stochastic discount factor valuation methods. The GPME method effectively allows for estimating betas for individual funds and portfolios and drawing statistical inference about performance (akin to regression methods with factor models). A drawback of the method is that it can be computationally demanding given the preferred estimation procedure based on the Generalized Method of Moments (GMM).

Another approach that relies on the selection of a public market benchmark is that of Excess Value (EV). As described in Turetsky et al. (2021), EV allows the comparison between the portfolio returns and the public benchmark in terms of currency as a measure of the actual dollar amount difference between the profit from the fund and the profit from the public benchmark. EV is especially useful for understanding total value changes and contributions. For example, EV provides a method that can be used for determining an alternative compensation structure for managers (i.e., distinct from carried interest) that depends on manager performance relative to a public benchmark.

2.7 Statistical methods for evaluating illiquid asset returns

There are a number of limitations when it comes to evaluating the returns of private investments. Given the nature of the assets, all returns rely on some form of estimation of current value, such as net asset value (NAVs). Substantial empirical evidence shows that NAVs are smoothed and systematically biased. Because it is often difficult to observe accurate returns, it is also hard to obtain reliable correlations between private assets and other assets. As discussed previously, another concern is that in practice private portfolios are not fully diversified so large sample statistics may not be relevant for a specific investor. It is very likely that portfolios with only a few private funds will have significant idiosyncratic risk.

A number of statistical methods have been developed to deal with the inherent difficulties of illiquid assets, and in particular, the unsmoothing of observed returns. Getmansky, Lo, and Makarov (GLM, 2004) provide a 1-step method that is effectively based on estimating a traditional moving-average time-series model of observed returns and then inferring the underlying process being averaged over time. The authors also provide a method for obtaining unbiased Sharpe Ratios for smoothed return series. In the next section we use the GLM method to unsmooth hedge fund index returns before conducting risk and return attribution analysis.

Couts, Gonçalves, and Rossi (CGR, 2020) discuss shortcomings of prior methods including the GLM method and propose a 3-step generalization of these methods which addresses issues related to spurious autocorrelations in aggregated data. For example, when the 1-step GLM method is used to unsmooth individual fund returns, aggregated indices of these funds will still exhibit significant autocorrelation (i.e., smoothness). The CGR method utilizes information on returns of peer funds to provide more complete unsmoothing.

These methods typically assume that there exists a time series of reported returns (e.g., for hedge funds), but for more illiquid fund structures such as private equity and direct private investments there may be very infrequent or especially unreliable periodic return data. Some other methods can be used to infer unbiased return series in these cases. For example, Brown, Ghysels, and Gredil (2021) estimate unbiased asset values at the weekly frequency using a state-space "now casting" model that uses fund cash flows, NAVs, industry returns, market returns and peer fund information as inputs. Gupta and Van Nieuwerburgh (2021) implement a "strip-by-strip" method that constructs a replicating portfolio using implied cash flows on listed equity and fixed-income investments. This method allows an asset pricing model to capture the risk in the cross-section of factors and provides a time-series of expected returns for each fund.

In general, unsmoothing has little effect on average returns, but can have substantial effect on estimated volatility and systematic risk exposures (e.g., an assets market beta). Thus, a proper model for generating unsmoothed returns is necessary for understanding risk-adjusted returns as well as the impact of an asset on overall portfolio risk. We now turn to examining performance in major alternative asset classes as specific examples of many of the methods described above.

3. Application and Results by Type of Asset

3.1 Liquid assets

There have been many studies considering the return profiles and risk attributes of liquid assets given that the basis for modern portfolio theory is rooted in these investments. For example, the examination of mutual funds goes back over 50 years to Sharpe (1966) who made the relationship explicit between capital market theory and various models of portfolio performance. Over the last few decades, the literature has demonstrated that mutual funds, on average, tend to underperform benchmarks largely because of fees. For example, Carhart (1997) demonstrates that persistence is almost entirely explained by common factors in stock returns and investment expenses. Berk and Green (2004) find that the lack of excess returns is consistent with a model of rational investors competing over scarce skills. Barber, Huang, and Odean (2016) find that investors use a number of factors when evaluating the skill of portfolio managers and that the more sophisticated the investor, the more sophisticated the benchmarks will be used in the evaluation process. Recently Song (2020) documents a mismatch between scale and skill in actively managed mutual funds which arises from investors crediting performance due to risk factor exposure to the talent of the portfolio manager. In addition Song finds that mutual funds often grow in size until they reach a point of significant underperformance based on poor allocation decisions by fund investors. Because performance analysis of liquid assets is well-established and straightforward, we do not discuss it further except for when it relates to analysis of alternative assets.

3.2 Semi-liquid assets

We define semi-liquid assets as those that cannot be immediately liquidated because of contractual terms (e.g., monthly or quarterly redemption rights as is common for most hedge funds) or assets that trade in markets where trading in size can take a long time (e.g., certain fixed income securities). Here, we focus on hedge funds since they represent a large class of semi-liquid assets and are owned by many institutional investors. We examine three main issues: First, what are the return properties of hedge funds after correcting for the statistical problems that arise from valuation smoothing? Second, how have hedge funds performed on a risk-adjusted basis using a multi-factor model (i.e., what are hedge fund alphas in a portfolio context)? Third, what has been the effect on common portfolio risk metrics when hedge funds are added to a diversified portfolio of public equities and bonds.

3.2.1 Hedge fund data

In our analysis, we use monthly returns for a variety of hedge fund indices from January 2005 through June 2021. While data on hedge funds are available prior to 2005, we focus on the more recent data because they are more representative of the fund universe currently available to institutional investors. This period contains several large market dislocations, including the great financial crisis, the European debt crisis, the "taper tantrum" of 2013, and the COVID-19 panic of 2020. We also examine two sub-periods: 2005-2012 (which includes the first two dislocations) and 2013-2021 (which includes the second two). Broadly speaking, the 2005-2012 period can be considered as a "bear" period of below-average market returns and the 2013-2021 period can be thought of as a "bull" period of above-average market returns. In this analysis, we only examine hedge fund indices and not individual funds. Specifically, we examine indices from six providers: BarclayHedge, Bloomberg, EurekaHedge, HFR, Morningstar, and PivotalPath. In much of our analysis, we focus on indices from Pivotal Path because we can observe the constituent funds. Barth, Joenvaara, Kauppila, and Wermers (2020) show that the other indices are unlikely to have some of the larger institutional-quality hedge funds, but these are generally included in the PivotalPath indices. We examine aggregate indices as well as indices for individual strategies. We note that investment professionals use these indices for relative performance comparisons. While the indices created by each hedge fund research firm are constructed in different ways with different assumptions, they each have the same goal of representing the overall performance available to investors.

Our risk analysis relies on benchmarks that employ a set of established market risk-factor series. All the series we examine are based on traded assets and so are conceptually available to sophisticated investors.[18] Appendix A provides a detailed description and source for each factor, so we simply summarize them briefly here. For primary asset class returns, we use the MSCI World Total Return Index as a proxy for global stock returns, the Bloomberg Barclays Global Aggregate Total Return Index as a proxy for global bond returns, and the S&P GSCI Index as a proxy for global commodity returns. We also include several standard risk factors: the Fama-French factors for small stocks (SMB), value stocks (HML), and momentum (MOM). Finally, we use the Pastor-Stambaugh (2003) illiquidity factor as a proxy for the premium earned by holding illiquid assets. While there are any number of other factors that could be considered, this set spans a wide range of factors risks and similar groups of factors have been examined in the recent hedge fund literature (e.g., Brown, Howard, and Lundblad, 2021).

Table 1 provides summary statistics for the aggregate hedge fund indices and market risk factors. The first part of Panel A shows statistics for raw monthly returns over the full sample period from 2005-2021. We observe meaningful differences in returns across the different indices. The Bloomberg index has the lowest average annual return of 5.16% and the PivotalPath index has the highest average annual return of 8.48%. This spread of 3.32% is more than half of the average return of all indices (6.42%). The annualized standard deviation of reported index returns (volatility risk) is more similar across indices ranging from a low of 5.08% for PivotalPath and a high of 6.88% for BarclayHedge.

[18] It has been noted that in practice some investors could have difficulty generating some of the risk factor return series because of institutional constraints. For example, many factors rely on short-selling or accessing low-liquidity stocks.

Table 1: Hedge Fund Data Summary

Data for the period 12/31/2004 - 6/30/2021 Returns and standard deviation are annualized

Panel A: Univariate Statistics

	Mean	*St Dev*	*Skew*	*Kurt*	*Auto-Corr*	*Mean*	*St Dev*	*Skew*	*Kurt*	*Auto-Corr*
Hedge Fund Indices (Annualized)	Raw Returns					Unsmoothed				
PivotalPath HF Composite Index	8.48%	5.08%	-1.04	4.49	0.26	8.46%	6.08%	-0.84	3.98	0.06
Bloomberg All HF Index	5.16%	5.98%	-1.30	7.27	0.12	5.33%	6.77%	-1.00	6.87	0.02
BarclayHedge HF Index	6.12%	6.88%	-1.26	4.80	0.24	6.11%	8.25%	-0.84	3.99	0.05
Eurekahedge HF Index	7.48%	5.21%	-0.68	2.93	0.21	7.47%	6.10%	-0.47	2.54	0.04
HFRI Fund Weighted Composite Index	5.71%	6.48%	-1.06	4.34	0.23	5.69%	7.64%	-0.78	3.73	0.05
Morningstar Broad HF Index	5.57%	6.47%	-2.03	13.07	0.15	5.54%	7.33%	-1.78	11.62	0.02
All Index Average	6.42%	6.02%	-1.23	6.15	0.20	6.43%	7.03%	-0.95	5.46	0.04
Market Factors (Annualized)										
Global Stocks	9.32%	15.38%	-0.75	2.25	0.12					
Global Bonds	3.36%	5.28%	-0.09	0.84	0.05					
Commodities	6.37%	23.59%	-0.71	2.53	0.23					
Small Stocks (SMB)	1.34%	8.31%	0.30	-0.21	-0.03					
Value Stocks (HML)	-2.48%	10.19%	-0.34	3.12	0.18					
Momentum	0.84%	15.66%	-2.39	16.41	0.15					
Illiquidity (Pastor-Stambaugh)	2.71%	12.89%	-0.27	5.27	0.07					

Panel B: Correlation Matrix

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
PivotalPath HF Composite Index (1)	1.00												
Bloomberg All HF Index (2)	0.96	1.00											
BarclayHedge HF Index (3)	0.94	0.99	1.00										
Eurekahedge HF Index (4)	0.97	0.99	0.97	1.00									
HFRI Fund Weighted Composite Index (5)	0.96	0.99	0.99	0.99	1.00								
Morningstar Broad HF Index (6)	0.88	0.83	0.82	0.87	0.86	1.00							
Global Stocks (7)	0.83	0.93	0.92	0.87	0.90	0.69	1.00						
Global Bonds (8)	0.30	0.33	0.31	0.35	0.33	0.25	0.36	1.00					
Commodities (9)	0.61	0.66	0.65	0.64	0.67	0.58	0.59	0.25	1.00				
Small Stocks (SMB) (10)	0.34	0.36	0.38	0.33	0.38	0.23	0.28	-0.09	0.24	1.00			
Value Stocks (HML) (11)	0.14	0.24	0.20	0.16	0.19	0.24	0.25	-0.05	0.22	0.15	1.00		
Momentum (12)	-0.28	-0.45	-0.35	-0.30	-0.33	-0.20	-0.40	-0.15	-0.19	-0.18	-0.45	1.00	
Illiquidity (Pastor-Stambaugh) (13)	0.47	0.49	0.48	0.46	0.48	0.43	0.33	0.02	0.49	0.30	-0.01	-0.02	1.00

Interestingly, Table 1 reveals no apparent relationship between index average returns and volatility risk. The composite indices are relatively similar in regard to skewness and kurtosis as well (with the exception of the kurtosis of the Morningstar index). All indices have slight negative skewness which represents more asymmetric downside risk than from the symmetric Gaussian normal distribution. Funds also have return distributions with fatter tails (positive kurtosis) meaning they are likely to have extreme positive and negative outcomes more frequently than if returns were normally distributed. Perhaps most importantly, all the indices exhibit significant autocorrelation of returns which is consistent with return smoothing at the broad index level.

To address the issue of smoothed returns, we apply the unsmoothing method of Getmansky, Lo, and Makarov (GLM, 2004) and report statistics for the unsmoothed index returns in the next part of Panel A of Table 1.[19] As expected, the results reveal higher smoothing-corrected standard deviations for all indices – by an average of about 1%, which is large compared to the average standard deviation of 7.03% – signifying that smoothed returns mask volatility risk. Other statistics also reveal some differences as a result of unsmoothing skewness is less negative and kurtosis drops slightly (suggesting adjusted returns are slightly closer to normally distributed). We also note that autocorrelations drop to close to zero (<0.06) suggesting that the GLM unsmoothing method we use is effective. This indicates that the more sophisticated 3-step unsmoothing method of Couts, Gonçalves, and Rossi (2020) is not needed in this application.

The final part of Panel A of Table 1 provides statistics for the market factors. We note that the global stock average annual returns of our sample period are about 2.8% greater than the average annual return across all the indices, but again there is quite a bit of difference across the indices. For example, the unsmoothed PivotalPath index has an average return that is just 0.86% less than global stocks. Global stocks are more than twice as volatile as all the unsmoothed hedge fund indices, yet skewness and kurtosis of global stocks is more like the hedge fund indices. Global bonds are both lower returning and less risky than all the hedge fund indices. Commodities earn less than most hedge funds but are much more volatile. The remainder of the panel shows statistics for the various risk factors. We point out a few noteworthy features. Value

[19] We use an MA(1) specification of the GLM model.

stocks underperformed growth stocks over this period. Momentum exhibits low returns, high risk, negative skewness and very high kurtosis over this period – meaning that it was a highly undesirable strategy over this period. The illiquidity factor earned almost 3% per year suggesting that compensation for bearing illiquidity risk was still significant over this period.

Of course, risk in a portfolio context is not just about volatility but is also determined by the correlation of returns across all assets in the portfolio. Panel B of Table 1 shows the contemporaneous Pearson correlation coefficients across all hedge fund indices and risk factors. Looking across row (7), we see that the HF indices are all highly correlated with global stock returns and commodity returns (about 0.9 and 0.6 on average, respectively). Hedge funds have positive but lower correlations with global bonds (about 0.3 on average). Other significant positive correlations are observed for small stock and illiquidity factors. Hedge funds have strongly negative correlations with momentum.

3.2.2 Hedge fund index returns

We now examine index returns and other statistics at different horizons for the various aggregate return indices. Table 2 reports returns, estimated alphas, volatilities and Sharpe ratios for horizons of 1, 2, 3, 5, 7, 10, and 15 years. Average returns over the 1-year and 2-year horizons have been above average for all the indices. However, returns for the 5-, 7-, 10-, and 15-year histories moderate substantially. We estimate alphas using a 7-factor model (global stocks, global bonds, commodities, along with size, value, momentum and illiquidity factors) and unsmoothed index returns. The estimated annual alphas vary significantly across indices and time horizons. For example, across different horizons, the estimated alphas are quite mixed across the various indices. Alphas for the 10-year horizon range from a high of 2.86% for the PivotalPath index to a low of -0.23% for the Bloomberg Index. These results suggest that assessing the effectiveness of hedge funds to help weather periods of market dislocation can depend greatly on the choice of hedge fund data.

Table 2: Comparison of Composite Hedge Fund Indices

Data as of 6/30/2021, returns unsmoothed following Getmansky, Lo, and Makarov (2004). Alpha from regression on Global Stocks, Global Bonds, Commodities, Small Stocks, Value Stocks Momentum, and Illiquidity. *1yr denotes trailing 12 months, 2yr denotes trailing 24 months, etc.

	*1yr**	*2yr*	*3yr*	*5yr*	*7yr*	*10yr*	*15yr*
Return (annual)							
PivotalPath HF Composite Index	20.06%	10.65%	7.86%	7.50%	6.09%	6.91%	8.18%
Bloomberg All HF Index	23.61%	11.19%	7.63%	7.05%	5.13%	5.33%	-
BarclayHedge HF Index	24.02%	12.12%	8.61%	8.41%	5.91%	5.83%	5.65%
Eurekahedge HF Index	22.04%	12.21%	8.89%	7.83%	6.35%	6.19%	7.03%
HFRI Fund Weighted Composite Index	27.40%	12.97%	9.23%	8.23%	5.80%	5.36%	5.26%
Morningstar Broad HF Index	26.59%	9.03%	6.63%	6.37%	5.23%	5.26%	5.16%
Alpha (annual)							
PivotalPath HF Composite Index	-1.46%	1.00%	1.25%	1.84%	1.81%	2.86%	4.49%
Bloomberg All HF Index	-2.77%	0.09%	-0.42%	0.01%	0.02%	-0.23%	-
BarclayHedge HF Index	4.25%	0.18%	-0.80%	0.68%	0.31%	0.22%	0.75%
Eurekahedge HF Index	-1.04%	1.25%	0.93%	1.33%	1.67%	1.78%	3.02%
HFRI Fund Weighted Composite Index	-2.79%	0.56%	0.58%	0.63%	0.43%	0.12%	0.61%
Morningstar Broad HF Index	-16.09%	4.37%	3.50%	1.70%	1.52%	1.51%	1.75%
Volatility (annual)							
PivotalPath HF Composite Index	6.94%	9.46%	8.35%	6.73%	6.10%	5.75%	6.19%
Bloomberg All HF Index	6.74%	10.44%	9.44%	7.57%	6.91%	6.77%	-
BarclayHedge HF Index	7.49%	12.23%	11.16%	8.86%	8.15%	7.87%	8.40%
Eurekahedge HF Index	6.53%	9.17%	8.23%	6.63%	5.98%	5.64%	6.12%
HFRI Fund Weighted Composite Index	8.21%	11.95%	10.89%	8.68%	7.86%	7.42%	7.77%
Morningstar Broad HF Index	8.94%	14.81%	12.35%	9.95%	8.81%	7.70%	7.49%
Sharpe Ratio							
PivotalPath HF Composite Index	2.88	1.05	0.80	0.96	0.87	1.11	1.17
Bloomberg All HF Index	3.49	1.01	0.68	0.79	0.63	0.71	-
BarclayHedge HF Index	3.20	0.93	0.66	0.83	0.63	0.67	0.56
Eurekahedge HF Index	3.36	1.26	0.93	1.02	0.93	1.00	0.99
HFRI Fund Weighted Composite Index	3.33	1.03	0.74	0.83	0.64	0.65	0.55
Morningstar Broad HF Index	2.97	0.56	0.44	0.53	0.51	0.61	0.56

Table 2 also reports hedge fund volatilities for various horizons. Historical volatility tends to follow similar patterns for the different indices, yet there are large differences in volatilities across at some horizons. At the 2-year horizon the volatilities of the Morningstar and Barclays indices jump much more than for the PivotalPath and Eurekahedge indices. Consistent with the variation in returns and volatility, the Sharpe ratios also vary significantly across indices as well as time horizon. For example, at the 5-year horizon Sharpe Ratios vary from a low of 0.53 for the Morningstar index to a high of 1.02 for the Eurekahedge index. Taken together, the results presented in Table 2 suggest that inference about the risk and return attributes of hedge funds will depend to a large extent on the dataset utilized. This dependency derives from the differences in constituent funds used to create the indices. Indices differ both in the set of funds they cover (e.g., with differing return and risk characteristics) as well as the composition (e.g., weighting) of sub-strategies in the broad indices. Consequently, we next undertake a more careful examination of sub-strategies.

3.2.3 Hedge Fund Sub-strategies

We conduct a risk analysis of hedge fund sub-strategies by estimating the 7-factor model on monthly data from 2005 to 2021. We do this for a variety of sub-strategy indices for the various index providers. For illustrative purposes, we provide detailed results for the PivotalPath sub-indices in Table 3.[20] The table reports estimated regression coefficients for the factor model along with other summary statistics. Coefficients that are statistically different from zero (at the 90% or better confidence level) are in bold text and asterisks denote the level of statistical significance.

There are several interesting results presented in Table 3. First, we note that the explanatory power of the factor model varies substantially across sub-strategies. As measured by the adjusted R-squared, the model explains 60% or more of return variation for 5 of the 8 sub-strategies. However, the model explains less than 40% of the return variation for the other 3 sub-strategies (quantitative equity, global macro, and managed futures). Second, different sub-strategies have different risk exposures. For example, the diversified equity index is significantly exposed to global stocks, small stocks, and illiquidity, whereas the managed futures index is significantly exposed to global bonds and momentum (MOM). Some risk factors are only significant for certain sub-strategies and not the composite index, and in fact, only global stocks, small stocks, and illiquidity are statistically significant for the composite. Third, the estimated intercepts of the models are almost always statistically positive indicating that the sub-strategies provided reliable excess returns on a risk-adjusted basis over this period. Annualized alphas implied by these intercepts range from 3.41% for event driven to 6.29% for equity sector specialists. Finally, we calculate information ratios based on the estimated alphas and volatility of the factor model residuals (idiosyncratic risk) and find substantial variation in these as well ranging from a low of 0.14 for managed futures to 0.49 for equity sector specialists.

[20] The appendix provides descriptions of these sub-strategies.

Table 3: Factor Exposures for PivotalPath Hedge Fund Indices

Data for the period 12/31/2004 - 6/30/2021, returns unsmoothed following Getmansky, Lo, and Makarov (2004). *,**,*** denote statistical significance at the 90%, 95%, and 99% confidence levels respectively.

	Composite	*Credit*	*Equity Diversified*	*Equity Quantitative*	*Equity Sector*	*Event Driven*	*Global Macro*	*Managed Futures*	*Multi-Strategy*
Global Stocks	**0.2789*****	**0.1714*****	**0.4676*****	**0.1357*****	**0.4343*****	**0.3793*****	**0.1542*****	0.0521	**0.1846*****
Global Bonds	0.0285	0.0197	-0.0869	0.0794	-0.0902	-0.0985	**0.1781****	**0.3535*****	0.0078
Commodities	0.0276	**0.0660*****	0.0128	-0.0037	0.0059	**0.0462***	0.0086	-0.0132	**0.0467*****
Small Stocks (SMB)	**0.0752*****	0.0218	**0.1367*****	-0.0359	**0.3085*****	**0.1422*****	-0.0273	-0.0295	**0.0518***
Value Stocks (HML)	-0.0442	0.0274	**-0.0834***	**0.0967****	**-0.1257****	-0.0771	0.0225	-0.0347	-0.0868
Momentum (MOM)	0.0079	**-0.0693****	0.008	**0.1294*****	0.0088	-0.0498	**0.0606****	**0.1536*****	-0.0279
Illiquidity (Pastor-Stambaugh)	**0.0670****	**0.1432****	**0.0677****	-0.0076	0.0477	**0.1356*****	0.0099	-0.0646	**0.1011*****
Intercept	**0.0035*****	**0.0039*****	**0.0033*****	**0.0036*****	**0.0051*****	**0.0028*****	**0.0029*****	**0.0031***	**0.0040*****
R-squared (adjusted)	0.74	0.63	0.86	0.26	0.83	0.76	0.24	0.12	0.61
Number of Obs.	192	192	192	143	192	192	192	192	192
Alpha (annualized)	4.28%	4.78%	4.03%	4.41%	6.29%	3.41%	3.54%	3.78%	4.91%
Information Ratio	0.39	0.33	0.37	0.34	0.49	0.22	0.21	0.14	0.39

Overall, the results presented in Table 3 suggest that the precise factor risk profile of a hedge fund portfolio will depend critically on its sub-strategy composition. This is important for at least two reasons. First, portfolio managers will need to do a careful analysis of their funds and strategy allocations to fully understand the risks to which they are exposed. Second, portfolio managers can deliberately construct hedge fund portfolios with factor risk profiles that complement risk exposures in other assets they own. Some allocators also take a market-neutral approach to hedge funds that attempts to isolate alpha, i.e., a portable alpha strategy. We discuss these more below.

It is interesting to compare sub-strategy returns and risk exposures across different index providers. However, this is difficult in some cases because the various providers do not have a standardized set of sub-strategy indices. To some extent, this creates an apples-to-oranges comparison problem. We now make comparisons for some strategies that appear to be similar across index providers: diversified equity, credit, event driven, global macro, multi-strategy, and managed futures across four index providers PivotalPath, BarclayHedge, Eurekahedge, and HFR (three providers in the case of credit). We repeat the factor model estimation for each index for each provider and then calculate the associated information ratios. We do this for the full sample period as well as the two sub-sample periods and report the results in Table 4.

As would be expected given the prior results, we document substantial variation in information ratios across sub-strategies, across index providers, and through time. For example, for the composite indices, the information ratios over the full sample period vary from about zero for BarclayHedge and HFR to over 0.3 for Eurekahedge and PivotalPath. While most of the information ratios are positive (because of generally positive alphas), it is almost always the case that values are lower for the 2013-2021 sub-period than for the 2005-2012 sub-period. This suggests that all sub-strategies have become less reliably beneficial to portfolios in recent years when markets have been experiencing higher returns. Indeed, the global macro and managed futures sub-strategies flipped from reliably positive contributors in 2005-2012 to largely negative contributors in 2013.

Table 4: Hedge Fund Sub-strategy Information Ratios

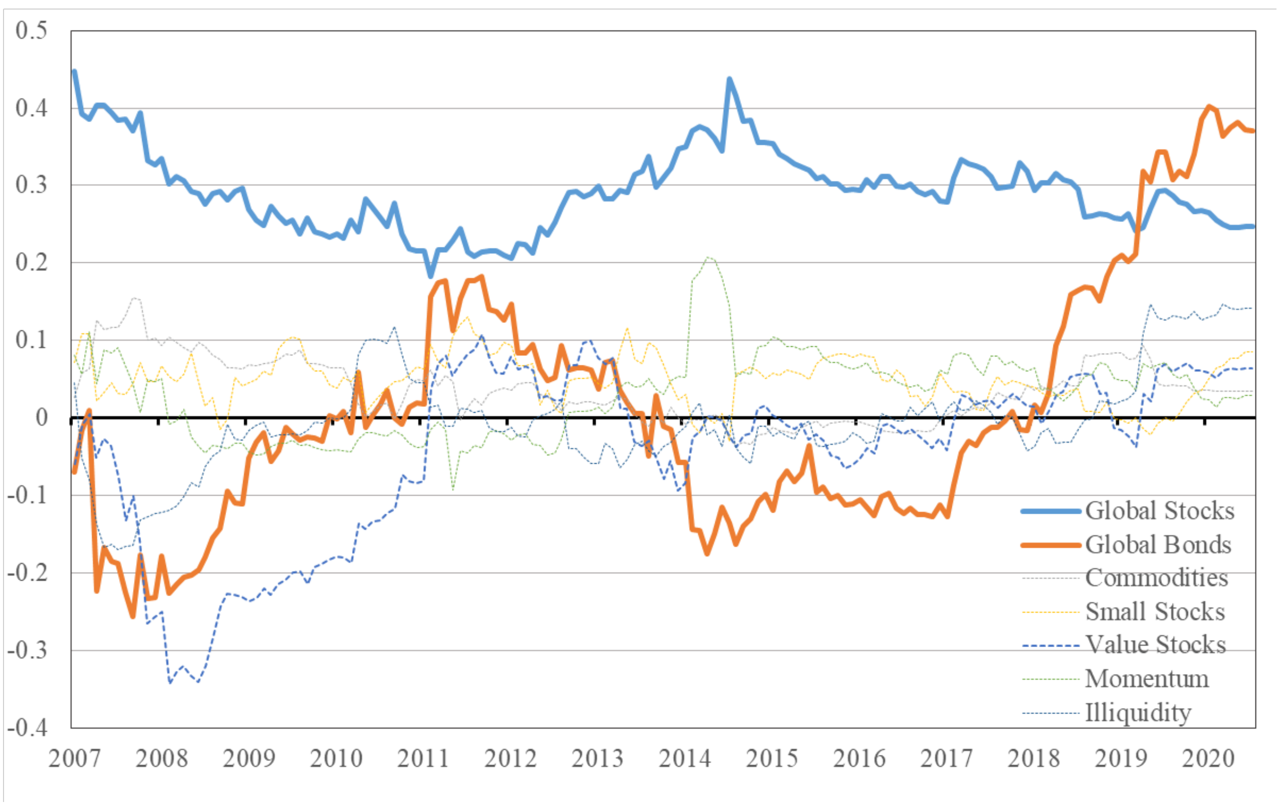
Data for the period 12/31/2004 - 6/30/2021, returns unsmoothed following Getmansky, Lo, and Makarov (2004)

	PivotalPath	*BarclayHedge*	*Eurekahedge*	*HFR*
Composite Index				
Full Sample	0.39	0.07	0.32	0.06
2005-2012	0.54	0.13	0.42	0.11
2013-2021	0.24	0.01	0.22	0.00
Equity Diversified				
Full Sample	0.37	0.10	-0.02	-0.01
2005-2012	0.56	0.14	-0.04	-0.02
2013-2021	0.16	0.05	-0.02	0.00
Credit				
Full Sample	0.33	-	0.28	0.20
2005-2012	0.47	-	0.42	0.26
2013-2021	0.27	-	0.20	0.19
Event Driven				
Full Sample	0.22	0.13	0.22	0.09
2005-2012	0.28	0.24	0.39	0.16
2013-2021	0.16	0.05	0.11	0.09
Global Macro				
Full Sample	0.21	0.08	0.06	0.04
2005-2012	0.30	0.12	0.09	0.10
2013-2021	0.06	-0.06	-0.07	-0.11
Multi-Strategy				
Full Sample	0.39	0.09	0.33	0.02
2005-2012	0.48	0.16	0.52	-0.10
2013-2021	0.32	0.01	0.16	-0.01
Managed Futures				
Full Sample	0.14	0.00	0.22	-0.08
2005-2012	0.20	0.04	0.32	-0.07
2013-2021	-0.01	-0.10	0.01	-0.16

overall exposure to global bonds, Figure 1 shows that the exposure varied from positive to negative throughout the sample period (solid orange line). These changes in exposure add an additional layer of difficulty for investors trying to understand their existing exposure risks in hedge funds as well as how to allocate in the future. Increasingly, services are being developed that allow investors to better understand these dynamics in real time through coordinated reporting of positions to third party risk aggregators such as Risk Metrics and SEI Novus.

Figure 1: Hedge Fund Composite Index Rolling Factor Exposures

Data for the period 12/31/2004 - 6/30/2021, 36-month rolling windows, returns unsmoothed following Getmansky, Lo, and Makarov (2004)



Ultimately, investors care most about how hedge funds contribute to overall portfolio risk-adjusted performance. Given the risk attributes we document, it is a difficult problem to determine the optimal mix of funds for a given portfolio. However, we can make a first pass at understanding the historical effects of adding hedge funds to a diversified portfolio of stocks, bonds, and commodities. We do this by considering a portfolio that initially consists of 60% global stocks, 30% global bonds, and 10% commodities. We then examine characteristics of portfolios that include an increasing fraction allocated to the PivotalPath composite index (while keeping the relative ratio between the market benchmarks constant).

Table 5 provides the results of this analysis for the full sample as well as the bear and bull sub-periods. We start by examining the full sample period and find that adding hedge funds consistently increases average annual returns and decreases portfolio volatility. Consequently, the composite portfolio Sharpe ratio increases from 0.61 for the case with no hedge funds to 0.83 for the case with a 40% allocation to hedge funds. There is very little change in portfolio skewness or Kurtosis. However, when we examine the two sub-periods, the effects are somewhat different. During the 2005-2012 sub-period, portfolio returns and risk are substantially improved by including hedge funds in the portfolio, thereby causing a substantial increase in Sharpe ratios. In the 2013-2021 period, however, the effect on returns from adding hedge funds is slightly negative though the effect on lowering volatility persists. The combined effect is still toward higher overall portfolio Sharpe ratios, but the improvement is smaller. In this later sub-period, the addition of hedge funds results in more negatively skewed portfolio returns and more overall tail risk (kurtosis) in contrast to the positive effects in the earlier sub-period. These results suggest that hedge funds have provided a consistent improvement in Sharpe ratios for diversified portfolios but this improvement is more consistently tied to a reduction in portfolio volatility than an improvement in portfolio returns. Together, these results suggest a need for additional research on the role of hedge funds in determining optimal asset allocation (e.g., allowing for a greater allocation to riskier assets with higher expected returns and the optimal use of leverage at the portfolio level to obtain optimal exposure to risk factors including those of hedge funds).

Table 5: Hedge Funds in Diversified Portfolios

Data for the period 12/31/2004 - 6/30/2021, returns unsmoothed following Getmansky, Lo, and Makarov (2004). Allocation to Hedge Fund represented by PivotalPath Composite Index

	Hedge Fund Allocation				
	0%	*10%*	*20%*	*30%*	*40%*
Full Sample					
Return (annual)	7.21%	7.33%	7.46%	7.58%	7.71%
Volatility (annual)	11.46%	10.83%	10.21%	9.60%	9.01%
Sharpe Ratio	0.61	0.66	0.71	0.76	0.83
Skewness	-0.86	-0.85	-0.84	-0.83	-0.83
Kurtosis	3.16	3.13	3.10	3.09	3.11
2005-2012					
Return (annual)	6.03%	6.41%	6.80%	7.18%	7.57%
Volatility (annual)	12.90%	12.14%	11.39%	10.65%	9.94%
Sharpe Ratio	0.45	0.51	0.58	0.65	0.74
Skewness	-0.97	-0.95	-0.93	-0.90	-0.88
Kurtosis	2.74	2.62	2.48	2.34	2.18
2013-2021					
Return (annual)	8.33%	8.21%	8.09%	7.96%	7.84%
Volatility (annual)	9.98%	9.49%	9.01%	8.54%	8.08%
Sharpe Ratio	0.80	0.83	0.87	0.90	0.94
Skewness	-0.50	-0.54	-0.59	-0.64	-0.69
Kurtosis	2.76	3.07	3.44	3.85	4.32

Overall, the results from the hedge fund analysis provide several take-aways:

- The various hedge fund indices we examine provide substantially different assessments of hedge fund risk and return attributes;
- All of the hedge fund indices suffer from significant return smoothing (autocorrelation) that needs to be addressed as part of performance analysis;
- Hedge funds provide exposure to a wide variety of risk factors and both the set of factors and their importance depend on the specific hedge fund sub-strategy;
- Hedge fund risk exposures can vary significantly through time;
- The relative benefits of hedge funds in terms of risk-adjusted performance and the contribution to diversified portfolios appear to have declined in recent years;
- Hedge funds do not appear to significantly reduce tail risk in portfolios (and may even exacerbate it in some periods).

3.3 Private equity and credit

In this section we present results from performance analysis and attribution for private equity and private credit funds. We analyze a large set of private funds in the Burgiss manager universe to determine the effects of benchmark selection with a specific eye to unpacking the role of geography, sector and size on performance estimates. We focus on direct alphas (DAs) as the performance measurement of choice, though examination of PMEs leads to similar conclusions. We then examine deal-level buyout performance using a "value bridge" approach applied to a sample of 2,951 fully-exited buyout deals provided by StepStone.

3.3.1 Fund-level benchmarking

We start the fund-level analysis by calculating DAs for the global sample of private equity funds in the Burgiss manager universe. We first consider all equity strategies (i.e., buyout, venture capital, expansion capital, and generalist) and then look at strategy types separately. We examine performance of all funds with vintage years of 1986-2016 through December 31, 2020. Estimates are based on pooled cash flows and beginning and ending net asset values.[21] We do not include more recent vintages because most funds are still in their investment periods and there have been relatively few exits compared to the more mature vintages. We examine historical returns for 3-, 5-, 10-, 15-, and 25-year horizons (all ending December 31, 2020).

Table 6 shows DAs using a variety of different benchmark indices. The first row indicates that when using the MSCI All-Country World Index (ACWI) the direct alphas have been in the range of 2-6% depending on which historical period is considered. There is not a clear pattern to the DAs with the most recent 3 years having performance similar to the 25-year performance.

An important benchmark characteristic is the size of companies in the index portfolio—and there has been a substantial debate in the literature as to whether it is better to use large-cap, mid/small-cap or all sizes as a benchmark. The MSCI-ACWI index is dominated by large companies and so the next two sections of Table 6 show DAs when the benchmark is restricted

[21] In effect, the calculations assumes that the portfolio of all funds is purchased at NAV values at the beginning of the evaluation period (e.g., December 31, 2015 for the 5-year performance statistics) and sold for NAV values on December 31, 2020. Between these dates, funds experience negative cash flows when making capital calls and positive cash flows when making distributions. Together theses NAVs and cash flows are used to calculate direct alphas.

to just mid-cap and small-cap stocks, respectively. The switch to a mid-cap benchmark has significant positive effects on DAs over more recent horizons and negative effects over longer horizons. Using a small-cap benchmark has similar, but somewhat more pronounced effects. Together these results suggest that the size of stocks in the benchmark is important, but the direction and magnitude of the effects will depend on the specific time horizon.[22]

Another drawback to using the MSCI-ACWI is that the style (e.g., value and growth) characteristics may not match that of the portfolio of private funds being evaluated. Table 6 also reports DAs separately for the MSCI-ACWI Value and Growth indices for both small and mid-cap benchmarks. At all horizons except 25-years the direct alphas using the value index are higher than for the growth index. The differences are especially large for shorter horizons. These suggest that matching on style can have a big effect on inference of market-adjusted performance. We also examine the importance of geography. The MSCI-ACWI index is a global index with the majority of it's market cap outside the U.S. for a large part of its history (though in recent years the U.S has moved to the majority). Global private equity has been shifting over the last 20 years from largely U.S. and U.K funds to a more global composition.[23] These facts suggest a possible geographic mismatch between the MSCI-ACWI and the pooled global PE portfolio. To gauge the potential importance of geography of the benchmark we calculate DAs using the Russell 3000, an index comprised of most actively traded U.S. stocks, and the MSCI-EAFE index, a commonly-used index of actively-traded stocks outside the U.S. Results presented in Table 6 show that there are large differences in DAs obtained from using these two indices as benchmarks with the Russell 3000 always resulting in lower DAs and differences trending up as the horizon shortens.

The results in Table 6 suggest that choice of benchmark has a very significant effect on inference regarding market-adjusted performance. We propose a method of creating custom benchmarks that match the geographic, sector, and size characteristics of the PE portfolio under consideration. We do this in the hopes of providing more accurate "apples-to-apples" comparisons for various fund strategies (and sub-strategies). In this analysis we consider custom indices based on sector and region (separately and together) and in future analysis plan to include

[22] We examined the distribution of sizes of portfolio companies in the Burgiss manager universe using the new Burgiss portfolio company dataset and concluded that the vast majority of private equity investments (both by number and total value) would be classified as small-cap stocks.
[23] See Aldatmaz, Brown, and Ansli-Kunt.

size matching as well.[24] The last three rows of Table 6 show the results. Matching the benchmark on sector, region, or both sector and region results in very similar DAs for the 15-year and 25-year horizons. In more recent years the differences are larger but the more important adjustment appears to be based on sector than region. The last row of the table shows that DAs using the custom benchmark based on sector-region weights provides results quite similar to (but slightly better than) those using the Russell 3000.

Table 6: Global Equity Fund Direct Alphas

Results through December 31, 2020. Sample includes all equity funds with 1987-2016 vintage years in the Burgiss master universe. Calculations use pooled cash flows net to LPs.

	Historical Excess Returns (Direct Alphas)				
Benchmark	3-year	5-year	10-year	15-year	25-year
MSCI ACWI	5.62%	2.38%	4.24%	4.53%	5.77%
MSCI ACWI Mid	8.02%	3.72%	4.81%	4.40%	4.87%
MSCI ACWI Mid Value	13.79%	5.79%	6.00%	4.85%	4.79%
MSCI ACWI Mid Growth	3.85%	2.15%	3.96%	4.13%	5.41%
MSCI ACWI Small	8.76%	3.28%	4.54%	3.31%	3.95%
MSCI ACWI Small Value	14.30%	5.75%	5.72%	4.05%	3.90%
MSCI ACWI Small Growth	3.90%	1.11%	3.52%	2.69%	4.07%
Russell 3000	2.02%	0.20%	0.23%	2.19%	3.81%
MSCI EAFE	11.59%	6.87%	7.60%	7.22%	7.74%
PE Sector-mix Index	2.13%	-0.23%	2.47%	3.29%	4.97%
PE Region-mix Index	4.70%	1.81%	3.13%	3.86%	4.94%
PE Sector-Region-mix Index	2.21%	0.27%	1.42%	2.89%	4.49%

Taken together the results presented in Table 6 provide evidence that benchmark selection is an important part of evaluating PE market-adjusted performance and that adjusting for the industry/sector composition of the portfolio under consideration is likely to be especially important. However, we note two important caveats regarding the reported performance statistics. First, our custom benchmarks rely on MSCI indices that are dominated by large cap stocks. Second, NAVs lag market returns and so the recent strong market-wide returns may not

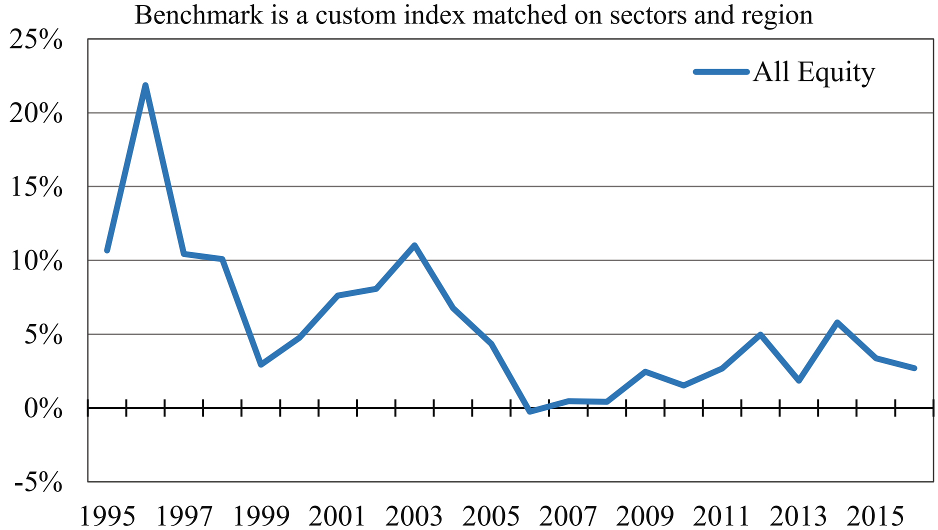
[24] For regions we use North America, Europe, Asia/Pacific, and Other. For sector, we use the GICS sectors. In all cases we utilize the MSCI suite of indices.

be fully reflected in the ending values we use. Both of these caveats suggest that recent performance numbers may be understated relative to values that reflect accurate NAVs and small stock returns.

We also examine direct alphas by vintage year using the custom benchmark matched on sector and region. We plot the results in Figure 2. DAs for the full sample of global PE funds were very high for 1995-1997 vintages due largely to the very high returns of venture capital funds. PE funds from 2001-2005 also had strong performance relative to public equity with strong performance from buyout funds driving the returns. 2005-2010 fund vintages have experienced much lower relative performance though only 2006 vintage funds performed worse than the sector-region matched benchmark. Funds with vintages since the GFC have DAs to date in the 2-5% range. Below we examine the trends by vintage year for buyouts and VC funds separately.

Figure 2: Direct Alpha by Vintage (Global Funds)

Benchmark is a custom index matched on sectors and region



It is also interesting to examine the performance of funds based on their investment region. In Table 7 we report direct alphas for equity funds for four regions: North America, Europe, Asia/Pacific (APAC), and All Other (primarily Africa, South America, and the Middle East). We examine a wide range of U.S. benchmarks including a sector-matched custom benchmark. For other regions we use our sector-matched benchmark and the appropriate MSCI index. For North America, we again see a substantial spread of DAs depending on which benchmark is used though in all cases the DAs are positive. The differences tend to be larger for more recent returns. Long-run (25-year) DAs are in the range of 3-5% for all benchmarks but near-term DAs (3-year) vary from 3.65% when using the Russel 2000 Growth index to as high as 15.72% when using the Russel 2000 Value index. The sector-matched index results in DAs in the range of 3-5% except for the 5-year horizon which has a value close to zero.

Table 7: Regional Equity Fund Direct Alphas

Results through December 31, 2020. Sample includes all equity funds with 1987-2016 vintage years in the Burgiss master universe. Calculations use pooled cash flows net to LPs.

	Historical Excess Returns (Direct Alphas)				
Benchmark	3-year	5-year	10-year	15-year	25-year
North America Equity					
S&P 500	4.61%	1.23%	1.41%	2.98%	4.65%
CRSP VW (US)	5.58%	1.74%	2.91%	3.43%	4.75%
CRSP EW (US)	9.91%	4.18%	6.25%	4.21%	3.13%
CRSP VW ex Largest Decile (US)	7.09%	1.58%	2.50%	2.16%	2.99%
Russell 3000	4.51%	1.14%	1.56%	2.87%	4.47%
Russell 2000	9.18%	3.05%	3.87%	3.50%	4.57%
Russell 2000 Value	15.72%	5.76%	5.76%	5.08%	4.57%
Russell 2000 Growth	3.83%	0.91%	2.25%	2.15%	4.88%
Russell Microcap	10.86%	4.41%	4.38%	4.91%	
PE Sector-mix Index	3.65%	0.02%	3.42%	3.58%	5.40%
Equity excluding North America					
MSCI EAFE	6.89%	5.05%	5.12%	6.21%	6.46%
PE Sector-mix Index	5.41%	3.94%	4.35%	5.60%	6.03%
European Equity					
MSCI Europe	6.92%	7.21%	5.32%	6.33%	6.90%
PE Sector-mix Index	5.48%	6.11%	4.99%	6.09%	7.00%
Only APAC Equity					
MSCI AC Asia Pacific	6.39%	1.91%	5.48%	6.51%	6.06%
PE Sector-mix Index	6.80%	3.62%	5.11%	7.09%	6.49%
Only Other Equity (Ex. NA and EU)					
MSCI EM	5.65%	-0.90%	6.91%	4.82%	3.68%
PE Sector-mix Index	5.43%	2.35%	3.65%	4.98%	4.76%

Looking beyond North America, Table 7 shows that DAs for the rest of the world are consistently in the 4-7% range using either the sector-matched index or the MSCI-EAFE index. Results for just European PE funds are similar (in a large part because most funds outside North

America are European). APAC funds also have performed better than both the MSCI-APAC index and the PE sector-matched index at all horizons. Results for “other countries” are also strong for most historical horizons and mostly independent of whether we use the MSCI-EM index or our sector-matched index (with only the exception of 5-year returns relative to the MSCI-EM which is slightly negative).

We now examine performance separately based on investment strategy. We create two sub-groups of global equity funds based on the Burgiss taxonomy: i) venture capital and expansion capital (e.g., growth equity) funds and ii) buyout and generalist funds. Direct alphas calculated using our region-sector-matched benchmarks are shown in Table 8. We create separate custom benchmarks for VC & Expansion and Buyout & Generalist that reflect the differences in sectors and geographies for these strategies. Historical DAs for VC & Expansion are quite variable ranging from -0.68% for the past 5 years to 6.70% for the past 3 years with a long-run (25-year) DA of 3.33%. In contrast, buyout fund historical DAs are always positive and less volatile, but the values trend down from 4.57% for the 25-year horizon to just 0.18% for the 3-year horizon. We note again that these may understate results since NAVs tend to lag market returns and recent market returns have been strong. Also, large caps have outperformed small caps recently and we are not matching on underlying portfolio company size.

Table 8: Venture & Buyout Equity Fund Direct Alphas

Results through December 31, 2020. Sample includes all equity funds with 1987-2016 vintage years in the Burgiss master universe split into two sub-groups: i) Venture Capital & Growth Equity, ii) Buyout and Generalist. Calculations use pooled cash flows net to LPs.

	Historical Excess Returns (Direct Alphas)				
Sub-strategy / Benchmark	3-year	5-year	10-year	15-year	25-year
VC & Expansion - Global					
PE Sector-Region-mix Index	6.70%	-0.68%	1.36%	1.56%	3.33%
Buyout & Generalist - Global					
PE Sector-Region-mix Index	0.18%	0.51%	1.42%	3.28%	4.57%

We also examine direct alphas by vintage year, by fund strategy using our custom sector-region-matched benchmarks. Figure 3 shows the exceptional returns from venture capital in the 1995-1998 vintage years. With the collapse of the dotcom bubble, VC fund performance also collapsed and DAs were close to zero or negative for vintages from 1999-2009. It has only been for vintages after the global financial crisis that DAs have again been consistently positive. Interestingly, buyout fund DAs exhibit a quite different pattern by vintage year (Figure 4). Relative returns were good for late-1990s vintages but improved to over 10% for the early 2010 vintages. DAs for vintages from 2006-2010 were close to zero, but post-GFC vintages have experienced better performance. Of particular note is the diversification that comes from investing in both VC and buyout funds of particular vintages. While there is still considerable variation by vintage year of the All Equity DAs, it is clear that vintages with relatively strong/weak VC performance do not line up exactly with relative performance of buyouts.

Figure 3: Direct Alpha by Vintage (Global Funds)

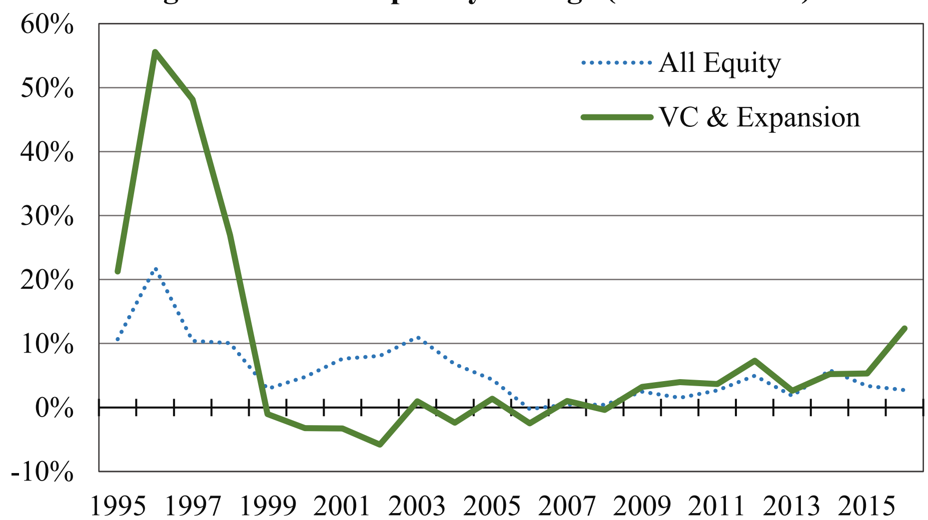
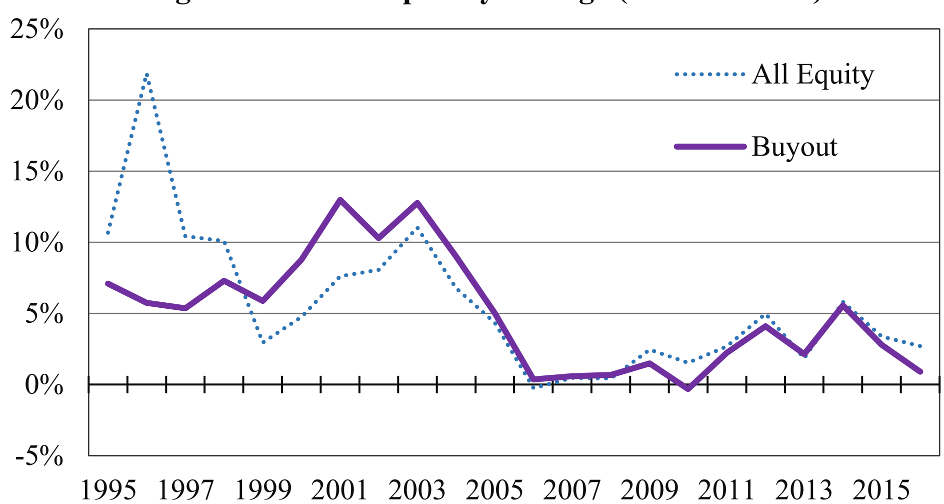


Figure 4: Direct Alpha by Vintage (Global Funds)



Finally, we examine performance of private credit funds of different types. Following Munday et al. (2018) we utilize the S&P/LSTA Leveraged Loan Total Return Index as the benchmark for direct alpha calculations. We also limit the time horizon under examination to the last 15 years because of data limitations. Results are presented in Table 9. We observe generally superior return of private credit for the 10-year and 15-year horizons across all strategies. However, for more recent years we find mixed returns relative to the benchmark. In particular, for the 3-year horizon all sub-strategies except Mezzanine funds have negative DAs with funds focusing on distressed assets performing worst. For the 5-year horizon values are close to zero for most sub-strategies. Again, we note that private fund NAVs may not fully reflecting the recent strong returns in public fixed income markets.

Table 9: Private Credit Fund Direct Alphas

Results through December 31, 2020. Sample includes all private credit funds with 1987-2016 vintage years in the Burgiss master universe as well as by sub-strategy. Calculations use pooled cash flows net to LPs. All results use the S&P/LSTA Leveraged Loan Total Return Index as the benchmark.

	Historical Excess Returns (Direct Alphas)			
Fund Type	3-year	5-year	10-year	15-year
All	-3.28%	-0.54%	2.81%	1.56%
Generalist (& Other)	-3.90%	-0.94%	3.04%	0.49%
Senior	-2.09%	0.28%	3.63%	1.97%
Mezzanine	0.19%	1.41%	3.86%	2.18%
Distressed	-5.26%	-1.67%	1.95%	1.49%

3.3.2 Portfolio company performance analysis attribution

Results above document the long-run outperformance of PE funds over public markets as well as the large variation in performance across PE funds. Key to interpreting historical performance is understanding how value is created at the deal-level during the time a company is owned in the fund portfolio. We define value creation as the change in the value of equity in the portfolio company between the time of the acquisition (entry) and the divestiture of the company (exit) by the private equity firm. During the holding period, managers aim to make their equity in the target company more valuable through a variety of strategies related to financial, operational, and governance engineering; the combination of which can result in improved revenue growth, margin improvements, additional free cashflow, optimized capital structure and ultimately higher valuations at exit relative to entry. However, some of the attributes associated with value creation can be viewed as more commoditized relative to others. For instance, capital structure today is infrequently a competitive advantage amongst private equity managers. While one manager may choose to use more leverage the pricing and terms are typically undifferentiated given a specific asset and capital structure. Conversely, the ability of a manager to realize consistent incremental value through operational improvements can be viewed as differentiated relative to its peers. As a result, the ability to ascribe and differentiate the attributes of value creation can provide valuable insights into manager selection and ultimately portfolio performance.

The conventional approach to analyzing equity value creation, namely the "value bridge," is to decompose the company's equity value appreciation into changes in three components: EBITDA, TEV/EBITDA ("EBITDA multiple"), and change in debt liabilities ("Net Debt"), so that

$$\Delta \text{ Equity Value} = \Delta \text{ EBITDA} \times \Delta \text{ EBITDA Multiple} - \Delta \text{ Net Debt.}$$

Over the life of the investment, the contribution of each component is quantified by the impact of the change in a specific component on equity value appreciation. Each value creation component can be expressed as a ratio of the company's equity value appreciation generated by that component over the company's equity value at entry. While this approach is simplistic in nature, it is broadly consistent with how a manager tends to create value and thus is commonly referenced by industry practitioners. However, the simplicity of the approach has several limitations. First, it does not fully account for the capital structure of the company, which prevents comparing investments with different leverage strategies due to the unequal financial risks. Second, this approach does not provide sufficient granularity to identify underlying drivers of value creation. For example, this approach does not distinguish EBITDA growth between revenue versus cost savings, market leverage versus incremental manager leverage or market changes in multiples from entry to exit. Furthermore, this simplified approach does not account for combination effects, resulting from the interactions of changes in one attribute affecting another. The effects of operating leverage and accelerated growth on valuation multiples are not accounted for. As a result, we utilize a more granular attribution framework.

In this study, we adopt Stepstone's Drivers of Investment Returns (DIR) framework, which addresses these shortcomings and provides measures of how GP-specific factors affect performance. We discuss the process in general terms here and provide specifics, including an example, in Appendix A. In contrast to the conventional value bridge, in which each value creation component is calculated to reflect its impact on the levered returns, the DIR framework calculates the contributions of value creation components on a "unlevered" basis. Specifically, the attribution of deal MOIC as if it is financed entirely by equity is used as a base. Then, the contribution from leverage or financial risk, is quantified separately by calculating how much additional equity appreciation, compared to the all-equity scenario, is generated using the actual leverage at acquisition. This decomposition not only quantifies the direct impact of financial leverage on equity appreciation, but also allows for comparison of different deals regardless of their leverage strategies at the time of acquisition. The unlevered return, measured by the growth of TEV during the holding period, is divided into two components, the EBITDA component and the EBITDA Multiple component. The EBITDA growth component is further split into two sub-components

$$\text{EBITDA} = \text{Revenue} \times \text{EBITDA Margin} .$$

EBITDA Multiples are frequently used as the unit of pricing in buyout transactions. GPs can create value by expanding the EBITDA Multiples through improving the growth, stability, and predictability of the portfolio companies. However, in addition to GP's efforts, multiple expansion can also result from the movements of market factors during the holding period. To identify the multiple expansion brought on by GPs, the EBITDA Multiple component is further decomposed into two sub-components, the Market Multiple and the GP Multiple. We compute the benchmark EBITDA Multiple of publicly traded companies for each industry and year pair. The EBITDA Multiple expansion of each portfolio company is compared to the change in the Market Multiple during the holding period based on industry classification. Any difference between the company's multiple expansion and the industry benchmark is attributed to the GP. For example, if a portfolio company's EBITDA Multiple expansion is less than the market benchmark multiple expansion, the GP Multiple component will be negative.

To analyze the GP's leverage decision at acquisition, the Leverage component is also decomposed into two sub-components, the Market Leverage and the GP Leverage. We compute the public benchmark of the leverage ratio (debt to TEV ratio) for each industry and year pair as the market benchmark. For each portfolio company, the Market Leverage component is computed as how much additional equity appreciation, comparing to the all-equity scenario, is generated by using debt that yields the same leverage ratio as the industry benchmark at acquisition. The rest of the Leverage component is attributed to GP Leverage component.

The last value creation component in the DIR framework is the Deleveraging component, which measures the impact of the change in net debt on equity appreciation during the GP's ownership. It is calculated as the ratio of the debt paydown (e.g., Net Debt at entry – Net Debt at exit) to TEV at entry. In total, there are six value creation components in the DIR framework, which sum to the equity growth during the holding period.

3.3.2 Buyout deal data

We utilize a new proprietary dataset of private transactions provided by the StepStone Group, which are derived from StepStone's investment due diligence process. This dataset includes information on a variety of deal characteristics, especially the key financial information (e.g., Net Debt, Revenue, EBITDA, EBITDA Multiple, cash flows, etc.) both at entry and at exit, which allows for the detailed value creation analysis. We limited our analysis to fully-exited buyout transactions.[25] We also required all deals to have values in Net Debt, Revenue, EBITDA, and EBITDA Multiple, both at entry and exit, which are necessary for our calculations. This resulted in a final sample size of 2,951 fully-exited deals from 1984 through 2018, with around \$945 billion USD in combined equity investments and around \$1.9 trillion USD in total enterprise value (TEV). By our estimates, these transactions cover about a quarter of the value of all (global) historical buyout activities with PE fund sponsors over this period.

In our sample, the deals were sponsored by 624 funds with an average fund size around \$1.7 billion, but there is a wide range of fund sizes represented. The typical deal is held for about 5.5 years with an interquartile range of 3 to 7 years. The funds typically take a majority stake in the buyout transaction, about 56% entry ownership on average. Values for entry TEV, Net Debt, Equity, and Revenue show that deal size is quite skewed with a relatively large number of small and mid-sized transactions in the dataset and a few much larger deals. For example, the mean entry TEV is \$665 million, which is greater than the 75th percentile breakpoint of \$501 million. These features are expected given the known composition of PE buyout transactions. The mean entry EBITDA Multiple is 8.25x with an interquartile range of 5.93x to 9.48x. Over the life of

[25] Various other screens are also applied. Fund size must be greater than 10 million USD, and deal entry TEV, exit TEV, and entry equity must be greater than 5 million USD. We similarly drop deals representing more than 50% of a fund's investment. In addition, we remove deals with MOICs greater than 20.

the deal, the average EBITDA Multiple increases by 1.77x with an interquartile range of -0.06x to 3.71x and the average deal MOIC is 3.67x with an interquartile range of 1.68x to 4.65x.

Table 10 provides some performance statistics for the sample. For the 2,951 portfolio companies in our sample, the weighted-average MOIC is 2.65x. We also calculate the unweighted mean and median of MOIC, which are 3.66x and 2.97x, respectively (results not tabled). As measured relative to the Burgiss buyout deal universe, the deals in our sample are better than average for two reasons. First, we select only fully-realized deals which biases recent deals toward more successful transactions (which are known to have shorter deal durations). Second, overall the StepStone data has somewhat better than average deals because some of the data is obtained through due diligence of previous funds which introduces a positive selection bias. Overall, as compared to the Burgiss quartile breakpoints, about 55% of the deals are top quartile, about 22% are second quartile, and 23% are below median. These multiples are based on the deal characteristics, not the GP's experience. If we calculate MOICs on cash flows invested and returned to the GPs (MOIC Invested), overall MOICs fall by about -0.33 to 2.32 and the experiences across quartiles compress toward the mean. Gross PMEs of the deals in our sample are also better than the average deal in the Burgiss universe. The mean PME is 1.85 with top quartile deals reaching 2.99. Below median deals still have average PMEs close to one (0.95) indicating that gross returns close to market returns.

Table 10: Buyout Deal Sample Statistics

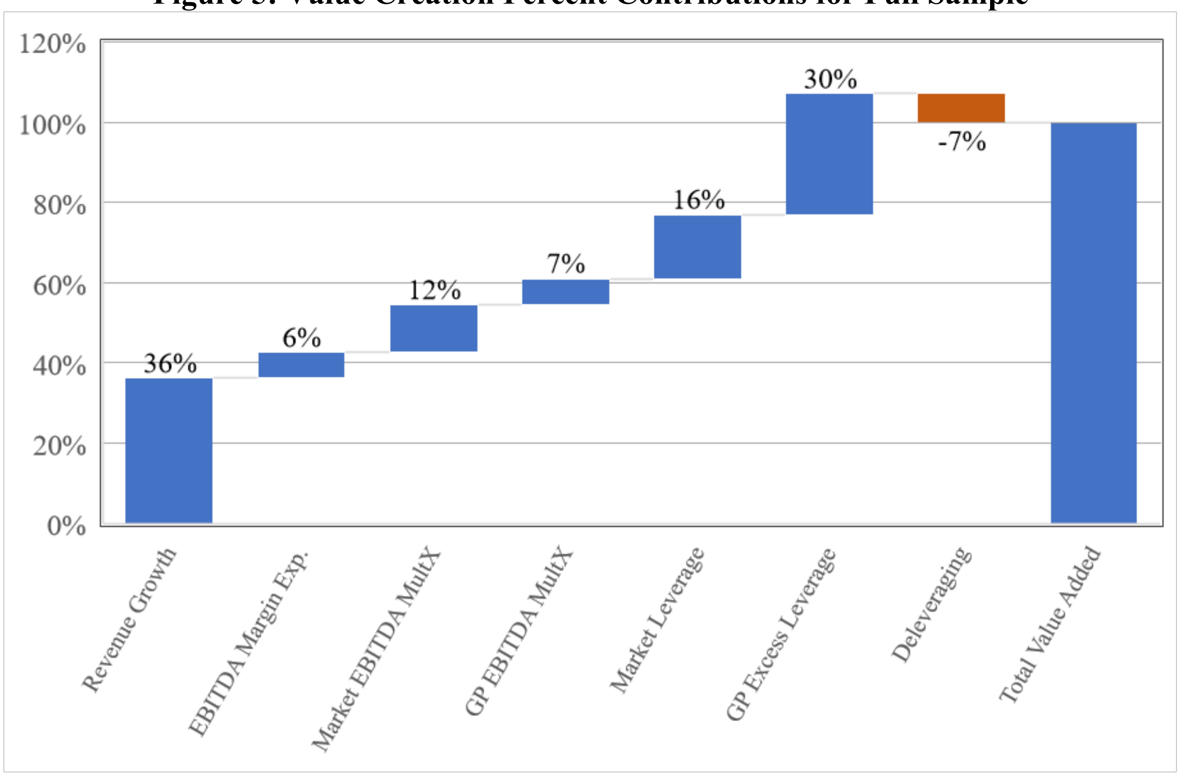
	Number of Obs.	MOIC	MOIC Invested	PME
Full Sample	2,951	2.65	2.32	1.85
By MOIC Quartile (Burgiss Breakpoints)				
Quartile 1 [>2.66]	1,620	4.95	3.63	2.99
Quartile 2 [1.56, 2.66]	655	2.03	2.13	1.59
Quartile 3&4 [<1.56]	676	0.87	1.18	0.95

In addition to Stepstone's deal-level data on private equity transactions, we utilize the Compustat data on financial statements of publicly traded companies to derive EBITDA Multiples and the Leverage Ratios benchmarks by industry. Changes in industry EBITDA Multiple benchmarks reveal the movements of key market factors, e.g., risk free rate, risk appetites, credit conditions, etc., influence the pricing of transactions. The benchmark Leverage Ratios show the leverage usage by sectors in each year and enable us to infer the extent that managers utilize financial leverage different than what is observed in public companies in the same industry.

3.3.3 Deal value-creation attribution results

We apply the value-bridge methodology described above to the transactions in our sample. For each transaction, we create estimates of the value creation components discussed above and in Appendix B. We create aggregate statistics of each value creation component using a weighted-average based on equity values at entry. We then calculate percentage contributions to over-all value creation to mitigate concerns about the positive performance bias in the sample. We choose entry equity for our weighted averages because it is economically relevant and less affected by skewness than other measures. We also examine the differences in value creation between regions, industries, and sub-periods.

Figure 5: Value Creation Percent Contributions for Full Sample



The value creation attribution for full sample is shown in Figure 5 and Table 11. In Figure 5, the blue boxes represent the average contribution (in percentage terms) for each value creation driver for the full sample. Table 11 provides the MOIC contributions by Burgiss performance quartile as well as the percent contributions for the full sample. The contribution from operational improvement, namely the EBITDA component, is the sum of Revenue Growth component (0.60x) and EBITDA Margin component (0.10x), which accounts for about 42% of total value created. While 0.3x (19%) of MOIC is contributed by an increase in EBITDA Multiple from entry to exit, 0.19x (12%) is the result of industry-wide expansion and 0.11x (7%) is attributed to the GP. Value-creation from leverage is 46% (0.76x incremental MOIC) with the majority of this (0.50x) due to GP excess leverage. The contribution from deleveraging is negative (-0.12x) in our sample which means that on average GPs are increasing the level of debt while owning the company. These contributions vary significantly across performance groups. In the top quartile group, all return drivers (except deleveraging) are meaningful contributors. In contrast, for the below-median deals almost all value drivers are economically small.

Table 11: Buyout MOIC Attribution and Percent Contributions

	Revenue Growth	EBITDA Margin Expansion	Market EBITDA Multiple Expansion	GP EBITDA Multiple Expansion	Market Leverage	GP Excess Leverage	Delev-eraging	MOIC
MOIC Contributions								
Full Sample	0.60	0.10	0.19	0.11	0.26	0.50	-0.12	2.65
By MOIC Quartile (Burgiss Breakpoints)								
Quartile 1 [>2.66]	1.22	0.39	0.30	0.31	0.60	1.35	-0.21	4.95
Quartile 2 [1.56 to 2.66]	0.51	0.01	0.21	-0.04	0.21	0.17	-0.04	2.03
Quartile 3&4 [<1.56]	0.06	-0.10	0.08	0.02	-0.02	-0.09	-0.08	0.87
Percentage Contributions								
Full Sample	36%	6%	12%	7%	16%	30%	-7%	100%

We analyze whether value creation drivers have changed over time by dividing our sample into three sub-periods based on deal entry year. The first period is from the beginning of our sample through 1999 (i.e., through the dotcom bubble) with 272 deals entered during this period. The second period, from 2000 to 2007, encompasses the period leading up to the GFC; 1,500 deals fall into this group. The third period is from 2008 through 2019 and contains 1,179 deals.

The results of the sub-period attribution are presented in Table 12. The last column shows that the average MOIC trends down slightly over these three sub-periods. However, more interesting are the trends for individual value-creation components. The results show a large decrease in the leverage component driven primarily by a decline in GP excess leverage from 46% in the pre-2000 subperiod to 22% in the most recent subperiod. Within the EBITDA component, the contribution from revenue growth is always the largest and quite stable in the mid-30%, but the contributions of other components shift over time. Specifically, contributions from both market and GP EBITDA multiple expansion have varied over the subperiods with market multiple expansion shifting from a modest headwind to a large tailwind over time. The EBITDA margin expansion is a consistent positive contributor but has become more important in the most recent subperiod. The contributions from leverage also show interesting patterns. In particular, market leverage has remained a nearly constant contributor in the range of 15-17% whereas the GP excess leverage contribution has fallen substantially from 46% in the pre-2000 subperiod to 22% in the most recent subperiod. Trends in deleveraging are also substantial with deals having paid down debt on average prior to 2000 and having increased debt in the post-GFC period.

Table 12: Value Creation Percent Contributions by Sub-period

	Revenue Growth	EBITDA Margin Expansion	Market EBITDA MultX	GP EBITDA MultX	Market Leverage	GP Excess Leverage	Delev-eraging	MOIC
All Years	36%	6%	12%	7%	16%	30%	-7%	2.65
Subperiods								
Pre-2000	36%	6%	-6%	-6%	15%	46%	9%	2.87
2000-2007	35%	2%	0%	12%	15%	37%	-2%	2.78
2008-now	37%	10%	25%	3%	17%	22%	-14%	2.52

The subperiod analysis reveals broad secular trends, however it masks well-known cyclical variation in buyout deal characteristics. To examine cyclical variation we examine attribution by entry year (results not tabulated). We observe low MOICs for deals entered right before the burst of dotcom bubble in 1999 and the GFC in 2008 driven largely by low contributions from leverage. Low values for the 2016 and later period are driven largely by a selection bias towards deals with quick exits. The highest MOICs are observed for deals entered into in 1995 and earlier where leverage is by far the largest contributor to performance. In contrast, the good performance for 2014 vintage deals was driven primarily by strong EBITDA growth. In general, the leverage component shows a downward trend over time, which is consistent with the sub-period results.

To study the difference in value creation in different geographies, we divide our sample into three regions, North America, Europe, and Others (primarily Asia but also Africa, Latin America, and Middle East). There are 1,624 deal located in North America, 1,093 in Europe, and 234 in other countries. The results of the analysis by geographic region are presented in Table 13. North America has the highest MOIC of 2.77x followed by Europe at 2.44x while the performance of deals in other countries is the lowest at 2.21x. Revenue growth is a large value-driver in all regions whereas EBITDA margin expansion has only occurred (on average) in North America and Europe. EBITDA multiple expansion has been a value driver in all regions, but is more skewed toward market expansion in North America and Europe than in other countries. Deals outside North America and Europe also have larger contributions from market leverage and less value creation generated by GP excess leverage. Deals in other countries also tend to pay down debt during the life of a deal instead of taking on more leverage. Overall, we find strong similarities between North America and Europe and weaker alignment in value creation contributions with the rest of the world.

Table 13: Value Creation Percent Contributions by Geographic Region

	Revenue Growth	EBITDA Margin Expansion	Market EBITDA MultX	GP EBITDA MultX	Market Leverage	GP Excess Leverage	Delev-eraging	MOIC
North America	39%	7%	11%	7%	15%	32%	-10%	2.77
Europe	25%	6%	16%	3%	16%	35%	-2%	2.44
Others	44%	0%	9%	19%	29%	-9%	9%	2.21

We also analyze value creation for nine industries and present the results in Table 14. We find significant differences in the value creation across the industries. The MOICs range from 2.02x for communication industry to 3.19x for health care industry. Despite these differences in overall value creation, the contributions of individual components are fairly stable across most industries. Revenue and GP excess leverage are consistently large contributors to performance.[26] Likewise, the EBITDA multiple expansion components are consistent contributors across almost all industries, though more modest in magnitude. There is evidence of different value creation strategies across industries. For example, information technology ranks in the middle of industries for overall performance yet it has the second highest EBITDA component, and ranks seventh in terms of MOIC because of the low leverage. The deleveraging contribution is negative for most of the industries in our sample indicating that debt expansion during a deal's lifespan is fairly common across industries.

Table 14: Value Creation Percent Contributions by Industry

	Obs.	Revenue Growth	EBITDA Margin Expansion	Market EBITDA MultX	GP EBITDA MultX	Market Leverage	GP Excess Leverage	Delev-eraging	MOIC
Communications	256	43%	0%	13%	5%	21%	30%	-12%	2.02
Consumer Discretionary	584	31%	1%	13%	5%	17%	33%	0%	2.80
Consumer Staples	215	24%	9%	6%	11%	17%	33%	0%	2.96
Financials	148	50%	-4%	-6%	31%	52%	-25%	1%	2.47
Health Care	370	43%	1%	17%	-1%	8%	39%	-7%	3.19
Industrials	626	25%	6%	6%	8%	13%	46%	-4%	3.09
Information Tech	422	38%	17%	19%	3%	9%	29%	-15%	2.54
Materials	221	17%	11%	6%	13%	13%	41%	-1%	3.14
Other	109	56%	-3%	7%	6%	21%	26%	-12%	2.59

Overall, our deal-level attribution analysis suggests that value creation is driven by a variety of factors. Some of these are market-wide phenomena and some are specific to the deal, and thus more likely to be representative of GP involvement. We leave to further research a more detailed analysis of attribution such as the analysis of whether value-creation style of individual GPs is persistent, cyclical, industry-specific, etc.

3.4 Real estate and real assets

Institutional investments in real assets have a long history. Banks and insurance companies have invested in real estate and precious metals for more than a century. Until the 1970s almost all institutional investments in real assets were direct holdings (e.g., buildings, gold, etc.). However, like the market for private equity investments, the market for real assets has

[26] The exception is GP excess leverage for financial which is negative. This is likely due to the market comparison set that includes highly levered regulated financial firms such as banks that are rarely targeted in buyout transactions.

developed private fund structures targeting institutional investors over the last 30 years. These included funds specializing in real estate, natural resources (especially oil and gas), infrastructure, and commodities (e.g., CTAs that formed after the creation of the Commodity Futures Trading Commission in 1974). While the emergence of institutional real asset funds occurred alongside the development of other fund strategies, the high inflation of the 1970s and early 1980s resulted in demand for assets that would hedge inflation risk in a diversified portfolio, and as a result, generated substantial interest in real assets.

In this section, we provide a brief overview of issues related to performance analysis of real assets and especially commercial real estate given its preeminent position in the real asset space. We consider both direct and indirect (e.g., fund) ownership because such a large fraction of real estate is owned directly by end investors. However, we also examine the recent performance of private real asset funds of various types.

3.4.1 Commercial real estate

Ghent, Torous, and Valkanov (2021) provide a detailed analysis of commercial real estate (CRE) as an asset class. This subsection draws heavily on their analysis to summarize some important aspects of the performance of public and private commercial real estate as well as to provide some additional insights specific to institutional investors. The heterogeneous nature of real estate and the fact that a particular property trades only infrequently has made it more difficult to adequately document and understand the pricing dynamics of commercial real estate. We also refer readers to Riddiough (2021) which provides a detailed analysis of CRE portfolios held by pension funds.

The natural starting point for benchmarking commercial real estate is REITs because REITs are publicly traded. The empirical REIT literature is voluminous, partly because of the data availability for this segment of the market. It is therefore of interest to know how representative the properties REITs own are of the universe of CRE. Table 15, reproduced from the data presented by Ghent (2021), shows how the properties purchased by REITs differ from those purchased by private investors. REITs concentrate their purchases in the retail segment of the market. They buy slightly larger and younger properties on average. There is no difference in the quality of properties bought by REITs and non-REIT investors. However, Muhlhofer (2013) points out that REITs select properties primarily based on their net rental income, rather than expected capital appreciation, because they are prohibited by law from holding properties primarily for resale. One of the requirements to be a REIT, for example, is a minimum holding period of four years.

Table 15: REIT and non-REIT CRE Purchases

Table reproduced from Ghent, Torous, and Valkanov (2021). Year Built is the year the property was built or is anticipated to be completed in the case or properties still under development. Units is the number of square feet in 1000s. Q-Score-National is the proprietary RCA measure of the quality of the property. Development takes a value of 1 if the property is under one year of age at the time of purchase. Office takes a value of 1 if the property is an office property; industrial and retail are similarly defined. The underlying data, presented in Ghent (2019), come from RCA and cover 39 US MSAs from 2001 to 2015.

	Obs.	Mean	Median	Std. Dev.	Min.	Max
Panel A: All Transactions						
Year Built	109,082	1978.3	1985.0	26.7	1111.0	2020.0
Price ($MM)	115,734	15.0	5.7	42.8	23.5	2,950.0
Units	115,734	106.8	53.0	172.5	0.6	5500.0
Q-Score-Local	97,593	0.51	0.51	0.29	0	1
Q-Score-National	97,593	0.57	0.59	0.29	0	1
Development	115,734	0.02	0	0.15	0	1
Office	115,734	0.33	0	0.47	0	1
Industrial	115,734	0.35	0	0.48	0	1
Retail	115,734	0.31	0	0.46	0	1
Panel B: REIT Purchases						
Year Built	9,584	1987.5	1990.0	20.2	1635.0	2016.0
Price ($MM)	10,356	$ 25.0	$ 11.2	$ 66.5	$ 112.5	$ 2,800.0
Units	10,356	158.6	98.1	214.0	1.2	4348.1
Q-Score-Local	7,982	0.58	0.60	0.28	0	1
Q-Score-National	7,982	0.56	0.57	0.27	0	1
Development	10,356	0.03	0	0.17	0	1
Office	10,356	0.27	0	0.44	0	1
Industrial	10,356	0.33	0	0.47	0	1
Retail	10,356	0.40	0	0.49	0	1

Panel A of Table 16 displays summary statistics for the returns on CRE indices from five data sources. The first three returns series are the National Council of Real Estate Investment Fiduciaries (NCREIF) Property Index (NPI), Real Capital Analytic's (RCA) Commercial Property Price Index (CPPI), and CoStar's Commercial Repeat Sales Index (CCRSI). These series represent the returns of portfolios of privately held CRE and aggregate unlevered property-level returns. The remaining two return series are from the National Association of Real Investment Trusts (NAREIT) and CRSP-ZIMAN and are two widely used REIT indices that cover publicly traded CRE. We do not adjust the REIT series for their use of leverage so that they are not directly comparable to the privately held CRE series.

Table 16: Summary Statistics on CRE Index Returns

Table reproduced from Ghent, Torous, and Valkanov (2021). The table displays summary statistics of the five most widely used commercial real estate indices discussed in the text. Panel A contains the results for total returns (TotRet), whenever available. In Panel B, we show the statistics for the income return (IncRet) and price appreciation (PrRet) parts. The following macroeconomic and finance variables are summarized in Panel C: CPI inflation (CPI INF), three-month Treasury bill yield (TB3M), 10-year Treasury bond yield (TB10Y), the appreciation of the Case-Shiller repeat residential real estate sales index (CS), the Gilchrist and Zakrajsek (2012) spread, and the return to the CRSP value-weighted index (VW Ret).

Panel A: Returns on CRE Indices

	NCREIF TotRet	CPPI-RCA TotRet	NAREIT TotRet	ZIMAN TotRet	CCRSI-COSTAR PriceRet
Mean	9	11.9	10.7	11.9	5.5
StdDev	4.2	5.2	17.4	16.5	5.1
AR(1)	0.782	0.937	0.061	0.095	0.661
Skew	-2.14	-1.6	-0.39	-0.81	-0.98
Freq	4	4	12	12	12
N	162	64	556	456	267
Start Year	1978	2002	1972	1980	1996
End Year	2018	2018	2018	2017	2018

Panel B: Income and Price Appreciation Returns on CRE Indices

	NCREIF IncRet	CPPI IncRet	NAREIT IncRet	ZIMAN IncRet	NCREIF PrRet	CPPI PrRet	NAREIT PrRet	ZIMAN PrRet
Mean	7.05	7.3	7.74	6.92	1.97	4.54	2.91	5
StdDev	0.65	0.3	1.43	0.96	4.09	5.26	17.26	16.43
AR(1)	0.989	0.951	0.088	-0.047	0.777	0.94	0.067	0.105
Skew	-0.37	0.77	4.98	0.89	-2.03	-1.6	-0.41	-0.81

Panel C: Macroeconomic Variables

	CPI_INF	TB3M	TB10Y	CS	GZ	VW Ret
Mean	3.45	3.49	5.83	3.69	1.81	10.69
StdDev	1.2	0.91	0.82	1.73	0.28	15.18
AR(1)	0.57	0.99	1	0.93	0.97	0.07
Skew	0.62	1.04	0.89	-0.91	2.39	-0.54
Freq	12	12	12	12	12	12
N	858	1016	785	377	524	651
Start Year	1947	1934	1953	1987	1973	1964
End Year	2018	2018	2018	2018	2016	2018

We provide statistics for total returns and, when the data are available, both their price appreciation and income components. We report these statistics for the entire sample period a particular series is available. The NCREIF and RCA series are available at a quarterly frequency while the remaining series are at a monthly frequency. Means and standard deviations are in annualized percentages.

The average return of privately held CRE is between 9.0% (NCREIF) and 11.9% (RCA). The difference of approximately 3% is not due to the different corresponding sample periods (see appendix) but rather may reflect a difference in the risk characteristics of these indices. In particular, the standard deviation of NCREIF returns is lower (4.2%) than that of the RCA returns (5.2%). Some of these differences, however, may reflect the fact that the CPPI is a repeat sales index while the NPI returns reflect the use of appraisals and exhibit smoothing as a result. By contrast, the average CoStar return is much lower because it does not include an income return component. For publicly held CRE, the average return is between 10.7% (NAREIT) and 11.9% (ZIMAN). In the common sample period (see appendix), the two indices have comparable average returns of about 12%. From Panel A we also see across all indices that CRE index returns are negatively skewed with total returns of the NPI being most negatively skewed.

Panel B of Table 16 decomposes CRE total returns into their income and price appreciation components. The income return component is similar across indices. At about 7%, income returns represent a significant fraction of total CRE returns. Income returns also exhibit low volatility with relatively little and, in most cases, positive skewness. These results also characterize the publicly traded NAREIT and ZIMAN indices. Given its relatively large size and little volatility, the income return component of total CRE returns is particularly appealing from a risk-return perspective.

In light of the significant search and other transaction costs present in the privately held CRE market, we see in Panel A that the first-order serial correlations (AR(1)) of the total returns of privately held CRE indices are high. Total returns of publicly held CRE indices, by contrast, have low first-order serial correlation, in the range of 0.04 to 0.06, reflecting the efficiency of public capital markets. The first-order serial correlation patterns of total CRE returns also characterize the first-order serial correlation patterns of their corresponding price appreciation components. The AR(1) coefficient is close to 1.0 for the income component of privately held CRE returns but close to zero for the income component of publicly held CRE returns.

Looking across the five CRE return series, the largest differences are between privately and publicly held indices. We summarize the differences as follows: (i) the average total return of CRE is in the range of 9% to 12% per year; (ii) publicly held CRE returns have higher volatility (iii) privately held CRE returns have large downside risk which makes it a riskier investment than suggested by its low variance; (iv) the income component of private and publicly held indices is about 7% and exhibits little volatility; (v) price appreciation accounts for 2% to 4.5% of total returns and is more volatile; and (vi) the serial correlation of privately held CRE returns is large and positive, capturing the significant frictions prevailing in that market. For publicly held CRE returns, by contrast, the serial correlation is close to zero as expected. To place CRE into a broader financial and macroeconomic environment, we now focus on one privately held CRE index (NCREIF) and one publicly held CRE index (NAREIT) and consider their relation to a variety of other financial and macroeconomic variables.

Panel C of Table 16 shows the summary statistics of the following six macroeconomic and finance variables: CPI inflation (CPI INF), three-month Treasury bill yield (TB3M), 10-year Treasury bond yield (TB10Y), the appreciation of the Case-Shiller repeat residential real estate sales index (CS), the Gilchrist and Zakrajsek (2012) credit spread and the return to the CRSP value-weighted index (VW). CPI inflation, the three-month T-bill yield, and growth in the Case and Shiller index have comparable averages, 3.45%, 3.49%, and 3.69%, respectively. The average GZ spread is 1.81%, the average 10-year Treasury bond yield is 5.83%, and the VW return is 10.69%. The macroeconomic series are all persistent with the exception of the inflation rate. The CS index exhibits a significant negative skew, similar to the CRE indices.

The average returns of the NCREIF and NAREIT indices exceed that of the 10- year Treasury bond and are comparable to the average value-weighted stock market return (VW). At first glance, the high average return and low variance of return to the NCREIF index might seem surprising. However, its large average return might be compensation for the negative skewness and significant downside risk in that portfolio. The CRE industry has traditionally classified properties into Core and non-Core types. For example, NCREIF defines Apartments, Freestanding Retail, Industrial, Office, Regional Malls, and Shopping Centers as Core property types while Health Care, Lodging-Resorts, Manufactured Homes, and Self-Storage are defined as non-Core property types (see, for example, Pagliari et al. (2005)). Investors often perceive Core property types as well as properties located in the Central Business District (CBD) of major markets to be less risky.

Table 17 examines the return properties of REITs focused on different property types. All the returns of core property types have higher means and standard deviations than the S&P 500. The average returns of core property types are all in the range of 10% to 15% annually with standard deviations ranging from 18% to 29%. Of the core property types, only Free-Standing Retail has a statistically significant alpha but is only significant at the 10% level. Industrial and Office have betas of 1. Apartments have a beta of 0.6 while Retail property types have betas ranging from 0.5 to 0.8.

Table 17: Monthly REIT Index Returns by Property Type, 1994-2018

Table reproduced from Ghent, Torous, and Valkanov (2021). Returns are annualized. For Alpha, * and *** denote statistically significant at the 10% and 1% levels for a two-sided test. Core and non-Core property type designations from Pagliari et al. (2005) which in turn are based on NCREIF classifications.

		Average	Std.Dev.	Beta	Alpha
Core					
	Apartments	12.6	19.4	0.64	0.39
	Free-Standing Retail	13.2	17.8	0.47	0.53*
	Industrial	14.1	29.4	1.00	0.34
	Office	12.3	20.9	1.00	0.34
	Regional Malls	13.8	25.2	0.81	0.41
	Shopping Centers	10.3	21.5	0.70	0.17
Non-Core					
	Healthcare	12.5	20.4	0.54	0.43
	Lodging-Resorts	9.4	29.7	1.21	-0.15
	Manufactured Homes	12.9	17.9	0.50	0.48*
	Self-Storage	16.4	19.5	0.51	0.77***
Other Assets					
S&P 500		10.0	14.9		
10-yr US Treasury		4.2	0.5		

Overall, the returns on REITs focusing on non-core properties do not indicate that non-core properties are any riskier than core properties. Furthermore, the returns of non-core properties may be less cyclical than those of core properties. Of the non-core property types, Health Care, Manufactured Homes, and Self-Storage all have betas of around 0.5 while Lodging has a beta of 1.2 consistent with vacation expenditures being highly cyclical. Lodging REITS have also returned an average of only 9% per year with a standard deviation of 30%. In contrast, Self-Storage has the highest average returns at 16.4% per year with a standard deviation slightly below that of most core property types. Furthermore, Self-Storage has a statistically significant alpha. However, the alpha is only 77 basis points per year. Finally, REITs of property types with high average returns tend to have low betas. This “betting-against-beta” anomalous behavior, which has been pointed out for non-REIT equities by Frazzini and Pedersen (2014), is particularly pronounced for non-core REITs. In particular, Lodging-Resorts has a large beta of 1.21 and a low average return of 9.4%, whereas Health Care, Manufactured Homes, and Self-Storage have betas of around 0.5 but their returns are 12.5% or higher.[27]

3.4.2 Other real asset funds

While real estate funds are the oldest and most plentiful type of real asset private fund, There are now substantial assets in other types of funds including natural resources and infrastructure. Table 18 reports summary statistics through 2021:Q3 for the Burgiss Manager Universe for funds classified as investing in real assets. The values represent sum totals of the number of funds and the capitalization in millions of US dollars for the full history for each fund type.[28] Burgiss tracks 1,340 real estate funds with about 917 million USD in capitalization. Within real estate, value-added funds make up the largest number of funds, however, opportunistic funds represent a larger share of capital. Burgiss tracks 376 natural resources funds, the majority of which are oil and gas related. These funds represent two-thirds of capital raised in natural resource funds. Timber and “other” strategies comprise most of the other funds in this group. Altogether, natural resource funds are less than 14% of total capital invested in real asset strategies. In recent years, infrastructure funds have rapidly grown in popularity. Burgiss tracks 281 infrastructure funds which is only about 14% of total real asset funds, yet because many of these funds are large, they represent about 31% of capitalization of real asset funds.

[27] See Van Nieuwerburgh (2019) for additional analysis of returns by property type.
[28] Burgiss defines capitalization as the total committed capital of a fund. Values are converted to USD for funds raised in other currencies using exchange rates at year-end of the fund vintage year.

Table 18: Burgiss Manager Universe of Real Asset Funds Through 2021:Q3

Fund Type	Number of Funds	Capitalization (Millions USD)
All Real Asset Funds	2,045	1,732.5
Real Estate Funds	1,340	916.9
Generalist	288	203.9
Value-Added	502	270.2
Opportunistic	401	386.0
Other	149	56.7
Natural Resources	376	235.8
Generalist	18	22.1
Oil & Gas	211	156.2
Timber	79	22.1
Other	68	35.4
Infrastructure	281	536.4
Generalist & Other	48	42.1

Real asset funds have experienced quite different trends in formation over the last 25 years. As shown in Figures 6 and 7, the overall number and capitalization of real asset funds have both grown substantially since the mid-1990s.[29] Growth, especially in real estate funds, accelerated in the mid-2000s before dipping substantially before and soon after the GFC. Examining trends in natural resource funds shows that the number and value of funds tended to increase until about 2015 at which point the popularity of these strategies reversed quickly so that by 2020 very few new funds were launching. In contrast, both the number and value of new infrastructure funds has grown quite rapidly since the GFC.

[29] Values for 2021 are only through Q3.

Figure 6: Number of New Funds

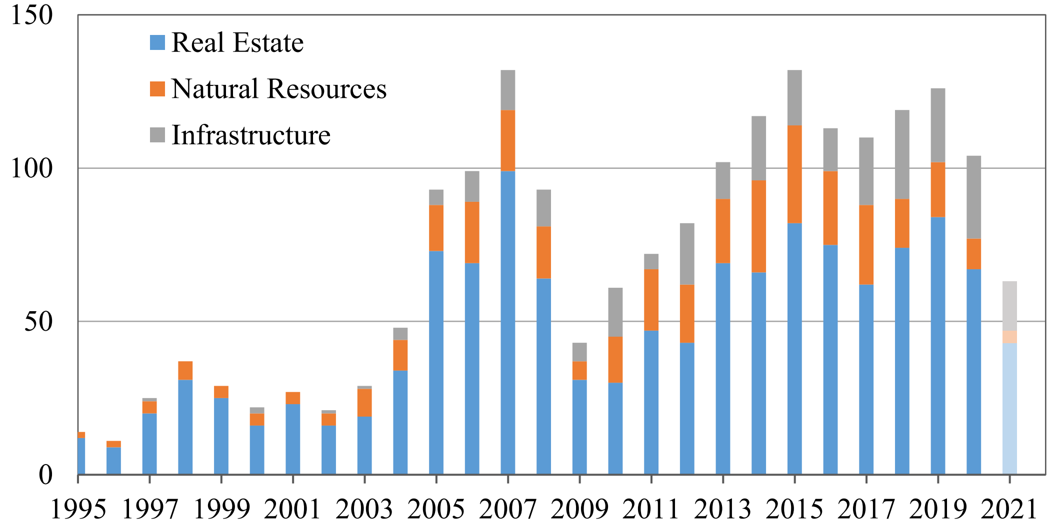
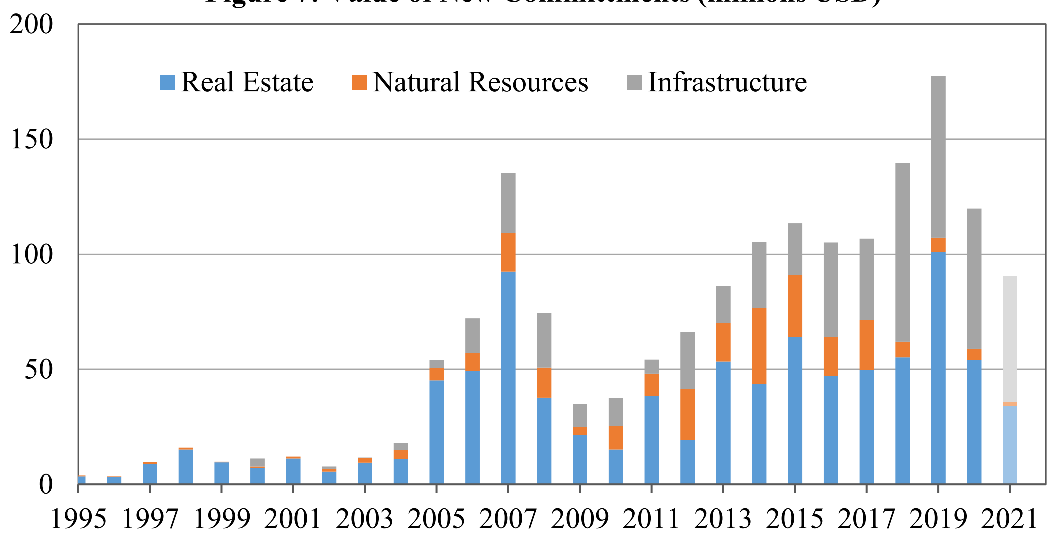


Figure 7: Value of New Committments (millions USD)



3.4.3 Direct holdings analysis

Much less is known about property-level returns than about index returns. Measuring property-level returns is difficult both because property NOIs are rarely reported and because of the scarcity of transactions. Ghent (2021) finds that only about 5% of the US CRE stock transacts in any given year. In contrast, turnover in the corporate bond market is about 50% annually according to Duffie, Garleanu, and Pedersen (2007). Because properties are heterogeneous and transact infrequently, analysts frequently use appraisal values to construct property-level returns. Unfortunately, for the same reason that returns themselves are difficult to measure in private CRE, appraisal values are often quite far from the actual price at which commercial property transacts. For example, Cannon and Cole (2011) find that appraisal values are, on average, 12% different from actual sales prices.

Sagi (2021) highlights the difficulties of measuring CRE returns on individual properties given the selection of which properties transact in a search model. What is often referred to as transaction risk constitutes one of the largest, if not *the* largest, source of risk in CRE investing. A further reason to analyze property-level returns is that, as Plazzi, Torous, and Valkanov (2011) show, exploiting property characteristics can improve performance of commercial property portfolios.

3.4.4 Fund-level return analysis

As noted above, performance analysis and benchmarking of real asset portfolios is especially challenging because of the very heterogenous nature of the underlying assets and the lack of high-quality data. In theory, we could create custom benchmarks (as we did for private equity) that adjust for type of asset, geography, and risk but unfortunately that type of analysis is not feasible for real assets with data currently available to us. We do note that, in general, real asset investors are seeking more persistent income and income growth with inflation pass-through. This typically results in lower betas and differentiation relative to private equity and public real asset indices. Here we present results of direct alpha performance calculations for components of the Burgiss manager universe using plausible public market benchmarks. As with the prior analysis, we assume that the betas of the private funds relative to the public benchmarks are 1.0 though some research suggests that real asset private funds may have much lower betas in some cases.[30] Results of the analysis are presented in Table 19 for 3-, 5-, 10-, and 15-year historical periods through December 31, 2020. We do not examine 25-year returns because the data are very sparse for many sub-strategies prior to 2005.

Results in Table 19 show that when benchmarked against the S&P Real Assets Total Return Index all real asset funds have performed poorly over the 3-year and 5-year history. Returns for the 10-year and 15-year histories have been closer to the public benchmark. One

[30] For example, an analysis by MSCI suggests that the beta of private infrastructure funds (after correcting for smoothed valuations) may be as low as half that of listed infrastructure. See, https://www.msci.com/www/blog-posts/assessing-private/02673389210.

challenge for the analysis is the difference in underlying assets between the benchmark and the funds. This shortcoming is mitigated to some extent by examining the real asset sub-strategies. The next section of Table 19 shows DAs for real estate funds calculated using the MSCI US REIT Index. Results for most horizons for most sub-strategies are negative suggesting private funds as a whole underperformed REITs. However value-added strategies outperformed the REIT benchmark for the 5-year and 10-year horizons. A shortcoming of this analysis is that the Burgiss manager universe includes global real estate funds whereas the benchmark includes only US REITs which have performed substantially better than non-US public real estate funds. As with private equity funds, recent returns may also suffer from conservative estimates of NAVs at the end of 2020 which are used as terminal values in the DA calculations.

Table 19: Real Asset Fund Direct Alphas

Results through December 31, 2020. Sample includes cash flows for all real asset funds with 1987-2016 vintage years in the Burgiss manager universe as well as by sub-strategy. Calculations use pooled cash flows net to LPs. Results use a variety of benchmarks as listed in the table.

	Historical Excess Returns (Direct Alphas)			
Fund Type: Benchmark	3-year	5-year	10-year	15-year
All Real Asset Funds:				
S&P Real Assets Total Return Index	-8.03%	-4.15%	0.72%	-1.72%
Real Estate: MSCI US REIT Index				
All Real Estate Funds	-3.44%	-0.15%	-0.44%	-3.86%
Generalist & Other	-3.82%	-1.49%	-1.22%	-4.41%
Value-Added	-0.52%	1.32%	1.13%	-3.01%
Opportunistic	-4.69%	-0.23%	-0.90%	-4.06%
Natural Resources (All Funds):				
Alerian MLP Total Return Index	-2.29%	1.63%	1.07%	-0.06%
MSCI-ACWI - Energy Sector	-4.65%	-4.08%	-0.42%	0.68%
Infrastructure (All Funds):				
MSCI World Infra Net TR USD Index	-8.24%	-2.88%	-0.35%	-0.27%
S&P Global Infra TR Index	-6.26%	-4.67%	-1.14%	-1.21%
DJ Brookfield Global Infra TR Index	-7.30%	-4.46%	-2.52%	-3.09%

For the natural resource funds, we calculate direct alphas with two benchmarks: the Alerian MLP Total Return Index and the MSCI-ACWI Energy Sector Index. Results using the MLP index suggest performance of natural resource funds in-line with MLP performance (+/- 2.5%) over all horizons. Results using the energy sector index suggest much weaker relative performance of around -4% for the 3-

year and 5-year horizons but with performance very similar to the index for 10-year and 15-year horizons. Finally, we examine infrastructure funds and calculate direct alphas using three different benchmarks: the MSCI World Infrastructure Net Total Return USD Index, the S&P Global Infrastructure Total Return Index, and the DJ Brookfield Global Infrastructure Total Return Index. The results are fairly similar across the three indices with substantial underperformance by private infrastructure funds for the 3-year and 5-year horizons and typically more modest underperformance for the 10-year and 15-year horizons. Here we again note possible shortcomings of the analysis relying on potentially downward-biased December 2021 NAV estimates and the assumed beta of 1.0.

4. Overall Portfolio Analysis and Attribution

The discussion in previous sections unveils the complexity of performance analysis and attribution in institutional portfolios that hold a mix of liquid, semi-liquid, and illiquid assets. In this section we discuss methods for better understanding risk and return properties of full portfolios, but readily admit that the science is relatively undeveloped. In the next section, we make suggestions for further research.

Brown, Ethridge, Johnson, and Keck (2021) provide a method for conducting performance attribution analysis for private fund portfolios. Their model uses an approach similar to the deal-level “value bridge” presented in Section 3.3.3. However, the analysis is done at the portfolio level and examines the attributes of fund allocations. Specifically, the model provides a method for estimating performance attributable to vintage year commitment timing, strategy section (e.g., buyout vs. VC funds), geographic allocation, commitment sizing and a residual component that includes fund selection. Results allow for comparison of a portfolio’s performance contributions along these dimensions relative to a benchmark portfolio of all possible private fund investments (e.g., the Burgiss Manager Universe). The analysis includes a historical simulation that generates approximate confidence intervals for each attribute for various portfolio types. The Brown, et al. (2021) model specifically considers portfolios of private funds, but the authors note that the model could be extended to include all asset types by including other assets in the benchmark portfolio. However, this could significantly complicate the analysis if the number of asset types is large (e.g., many types of liquid, semi-liquid, and illiquid assets).

Prior research such as Gupta and Van Nieuwerburgh (2021) and Goetzmann, Gourier, and Phalippou (2019) confirm that there are important systematic return components to private fund returns, but also that there also exist idiosyncratic returns in private funds that provide diversification benefits. The analysis in Section 3.2 provides similar evidence for hedge funds. Here we discuss a straightforward way to extend a factor-based approach to portfolio analysis that can provide insights into historical portfolio performance analysis as well as a mechanism for examining risk exposures, asset allocation, and expected returns.

We consider a hypothetical investment universe with just four traded systematic return factors: equity, inflation, real fixed income, and illiquidity. There are 3 primary asset classes: equity, fixed income, and real assets. Each of these has three sub-asset classes. For example, equity and fixed income have separate categories for public (liquid), hedge fund (semi-liquid) and private (illiquid) assets. There is also a fourth category for "diversifying assets" that may not map easily into the other three primary asset classes. The model assumes each asset/sub-asset class has expected returns and risk deriving from a factor structure. For our hypothetical example, we assume the following expected returns and standard deviations of the factors:

10-year Horizon	Equity	Inflation	Real Fixed Income	Illiquidity
Expected Returns	6.0%	3.0%	1.0%	3.0%
Standard Deviation	16.0%	2.0%	3.0%	13.0%

so that the equity factor has high expected returns and high risk, inflation and real fixed income have low expected returns and low risk, and illiquidity has low expected return and high risk. We assume for simplicity that the correlations between all factors are 0.2. We do not claim that these are realistic assumptions, instead this exercise is meant only to provide a tractable example. Each asset sub-class also has an excess return (α) and idiosyncratic risk which are assigned by the modeler.

The framework specified above is sufficient for determining the expected return and risk characteristics of a portfolio. We provide an example of one such portfolio in Table 20. The column labelled *Wt.* provides the sub-asset class portfolio weights. Assumed exposures to each of the four factors are in the columns labelled *β(.)* and when combined with assumptions for α provide expected returns in the column labelled *Total E[R]*. Assumptions for idiosyncratic risk for each sub-asset class are in column labelled *Idio*. Total risk (volatility) is the standard deviation based on the portfolio properties and used to calculate Sharpe Ratios (based on total returns, <u>not</u> returns in excess of the risk-free rate).

Table 20: Hypothetical Diversified Portfolio in a Factor-Model Framework

		Factor Loadings and Excess Return					Total	Risk (Std.Dev)		Sharpe
	Wt.	β(Eq)	β(Infl)	β(R)	β(Illiq)	α	**E[R]**	Idio	Total	**Ratio**
Equity										
Public Equity	15%	1.00	0.30	0.30	0.00	0.00%	7.20%	5.0%	21.2%	0.34
Equity Hedge Funds	5%	0.50	0.20	0.20	0.30	1.00%	5.70%	10.0%	19.4%	0.29
Private Equity	15%	1.20	0.10	0.10	0.50	2.00%	11.10%	15.0%	35.9%	0.31
Fixed Income										
Public Fixed Income	15%	0.00	1.00	1.00	0.00	0.00%	4.00%	5.0%	8.8%	0.46
FI Hedge Funds	5%	0.10	0.30	0.30	0.30	1.00%	3.70%	10.0%	14.7%	0.25
Private Credit	10%	0.30	0.30	0.30	0.50	2.00%	6.50%	15.0%	23.7%	0.27
Real Assets										
Real Estate	15%	0.70	0.30	0.30	0.20	0.00%	6.00%	10.0%	22.0%	0.27
Infrastructure	5%	0.50	0.50	0.50	0.30	1.00%	6.90%	15.0%	24.7%	0.28
NR & Other RA	5%	0.30	0.50	0.00	0.40	2.00%	6.50%	20.0%	27.6%	0.24
Diversifying Strategies	10%	0.20	0.20	0.20	0.20	3.00%	5.60%	15.0%	19.5%	0.29
Full Portfolio	**100%**	**0.56**	**0.38**	**0.36**	**0.24**	**1.05%**	**6.60%**	**11.0%**	**21.0%**	0.31

The results in Table 20 provide an intuitive way of parsimoniously considering the properties of a complex portfolio. For example, the last row shows the full portfolio factor exposures, expected α, total expected return and risk level. The portfolio is estimated to have an expected return of 6.60% and a total risk of 21% which equates to a Sharpe Ratio of 0.31. The portfolio is expected to have a positive alpha of about 1%, but this comes at the cost of substantial idiosyncratic and total risk associated with hedge funds, private assets, real assets, and diversifying strategies.

In theory this framework can be utilized to evaluate asset allocation decisions across sub-asset classes and solve for optimal allocations based on various assumptions. In the context of performance analysis and attribution, the model provides a framework for understanding how historical portfolio performance can be decomposed into market-wide factor exposures and excess returns and, importantly, how to relate this performance to market-wide and idiosyncratic risk components. This analysis can be conducted by estimating factor exposures on a historical basis and then calculating excess returns and idiosyncratic risk by sub-asset class. Of course, when using a model of this type in practice the usual caveats regarding the need for careful exposure estimation apply (e.g., unsmoothing of returns for semi-liquid and illiquid assets). In summary, the factor exposure framework provides a parsimonious and intuitive approach for understanding complex portfolios, but requires careful consideration of the right factor structure and parameter estimation.

5. Conclusions and Future Research

This paper has provided a summary of current research and thinking on how to conduct performance analysis and attribution for portfolios that include a wide range of assets commonly owned by institutional investors. Our updated analysis for hedge funds, private equity and credit funds, and real assets highlights the complexities of the problem facing asset managers and investors seeking to better understand portfolio performance drivers. Perhaps the most obvious takeaway from our analysis is the need for additional research. For example, we propose the following as potential topics for further investigation:

- Our private fund analysis assumes a beta of 1.0 to the benchmark with the admission that this is unlikely to be accurate for most strategies and funds. Future research should more reliably identify the appropriate beta for specific strategies and time periods.
- The factor-model approach would benefit from a deeper understanding of the risk factors most relevant to portfolios with alternative assets. An agreed-on set of factors would allow for easier analysis across sub-asset classes. This framework could then be better utilized for building optimal portfolios and examining issues related to portfolio-level leverage, optimal alpha capture, filtering through the "factor zoo" and the relevance of new factors (e.g., those potentially related to ESG investing).
- Hedge fund research continues to suffer from data quality issues. Future research should attempt to assemble a more comprehensive database of "institutional quality" hedge funds that aligns with the opportunity set of actual investors.
- Deal-level analysis of private equity transactions should further decompose returns into industry effects and deal-specific effects. For example, research can examine the effects of industry-wide changes in revenue and margin expansion.
- Evidence suggests an important role for an illiquidity premium in alternative assets. However, the use of measures based on public market returns like the Pastor and Stambaugh (2003) method may not be appropriate for private markets. Future research

should examine alternative measures of liquidity and estimates of illiquidity exposure and risk premia.

Ideally, we would like to move toward a commonly accepted framework and providing a "How To" implementation guide for CIOs and allocators that rests on a deeper understanding of alternative assets in a portfolio context. A shared framework could be beneficial to many market participants and incorporate existing approaches (e.g., a risk budget approach) as well as address other common questions such as the appropriate use of leverage and financial derivatives in optimal asset allocation

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Appendix A. Hedge Fund and Market Risk Factor Data Descriptions

Hedge Fund Data	
BarclayHedge	BarclayHedge Hedge Fund Index, BarclayHedge Equity Long/Short Index, BarclayHedge Event Driven Index, BarclayHedge Global Macro Index, BarclayHedge US Managed Futures Industry BTOP50 Index, BarclayHedge Multi Strategy Index
Bloomberg	Bloomberg All Hedge Fund Index
EurekaHedge	Eurekahedge Hedge Fund Index, Eurekahedge Structured Credit Hedge Fund Index, Eurekahedge Equity Long Short Fund of Funds Index, Eurekahedge Event Driven Hedge Fund Index, Eurekahedge Global Macro Fund of Funds Index, Eurekahedge CTA / Managed Futures Hedge Fund Index, Eurekahedge Multi-Strategy Hedge Fund Index
HFR	Hedge Fund Research HFRI Fund Weighted Composite Index, Hedge Fund Research HFRI Credit Index, Hedge Fund Research HFRI Equity Hedge Total Index, Hedge Fund Research HFRI Event-Driven Total Index, Hedge Fund Research HFRI Macro Total Index, Hedge Fund Research HFRX Macro/CTA Index, Hedge Fund Research HFRI EH Multi-Strategy Index
Morningstar	Morningstar Broad Hedge Fund Index
PivotalPath	Composite, Credit, Equity Diversified, Equity Quantitative, Equity Sector, Event Driven, Global Macro, Managed Futures, Multi-strategy
Market Factor Data	
Global Stocks	MSCI World Total Return Index. Source: Bloomberg
Global Bonds	Bloomberg Barclays Global Aggregate Total Return Index. Source: Bloomberg
Commodities	S&P GSCI Index Spot. Source: Bloomberg
Small Stock Factor (SMB)	Small Minus Big provides a measure of exposure to the premium paid for smaller company stocks outperforming large company stocks. Source: Fama/French U.S. Research Returns Data
Value Stocks (HML)	High Minus Low provides a measure of exposure to premium paid for value. Spread between companies with high book-to-market value ratios compared to low book-to-market value ratios. Source: Fama/French U.S. Research Returns Data
Momentum	Measures exposure to the persistence phenomena in which stock that have performed well in short-term past will continue to outperform in the near-term and vice versa. Buy winners, short losers. Source: Fama/French U.S. Research Returns Data

Appendix B. Value-Bridge Analysis

The unlevered return, measured by the growth of TEV during the holding period, is divided into two components, the EBITDA component and the EBITDA Multiple component.[31] The potentially problematic allocation of the combination effect in the conventional value bridge is remedied in the DIR framework by quantifying the combination effect explicitly and distributing it evenly across the two value creation components. More specifically, the combination effect of the EBITDA growth and the EBITDA Multiple expansion (Combo[EBITDA, EBITDA Multiple]) is calculated as the product of the EBITDA growth and the EBITDA Multiple expansion. The EBITDA component is then computed as the growth of EBITDA during the holding period plus half of Combo[EBITDA, EBITDA Multiple]. Similarly, the EBITDA Multiple component is calculated as EBITDA Multiple expansion during the ownership plus half of Combo[EBITDA, EBITDA Multiple]. While the assignment of 50% weighting is arbitrary, actual weighting can be refined based on comparable company public market data.

To provide deeper insights on how GPs improve portfolio company's operations, the EBITDA growth component is further split into two sub-components

$$\text{EBITDA} = \text{Revenue} \times \text{EBITDA Margin} \quad .$$

The same issue of combination effect arises here between Revenue growth and the EBITDA Margin expansion. (Combo[Revenue, EBITDA Margin]) is computed as the product of the Revenue growth and the EBITDA Margin expansion. Theoretically, Combo[Revenue, EBITDA Margin] should be attributed based on operating leverage of the portfolio company. For simplicity, we follow Stepstone's approach and distribute Combo[Revenue, EBITDA] evenly across the two sub-components. The half of Combo[EBITDA, EBITDA Multiple] inherited from EBITDA components is also distributed evenly across the two sub-components. The Revenue component is then computed as the Revenue growth plus a half of Combo[Revenue, EBITDA Margin] and a quarter of Combo[EBITDA, EBITDA Multiple]. Similarly, the EBITDA Margin component is calculated by EBITDA Margin expansion plus a half of Combo[Revenue, EBITDA Margin] and a quarter of Combo[EBITDA, EBITDA Multiple].

[31] A detailed example is shown in Table A1.

The EBITDA Multiples are usually used as the unit of pricing in buyout transactions. GPs can create value by expanding the EBITDA Multiples through improving the growth, stability, and predictability of the portfolio companies. However, in addition to GP's efforts, multiple expansion can also result from the movements of market factors during the holding period. To identify the multiple expansion brought on by GPs, the EBITDA Multiple component is further decomposed into two sub-components, the Market Multiple and the GP Multiple. We compute the benchmark EBITDA Multiple of publicly traded companies for each industry and year pair (Market Multiple [Industry = N, year = X]) as the ratio of the total TEV of Industry N to the total EBITDA of Industry N in Year X. The EBITDA Multiple expansion of each portfolio company is compared to the change in the Market Multiple during the holding period based on industry classification. Any difference between the company's multiple expansion and the industry benchmark is attributed to the GP. For example, if a portfolio company's EBITDA Multiple expansion is less than the market benchmark multiple expansion, the GP Multiple component will be negative. The Combo[EBITDA, EBITDA Multiple] inherited from the EBITDA Multiple component is distributed proportionally.

To analyze the GP's leverage decision at acquisition, the Leverage component is also decomposed into two sub-components, the Market Leverage and the GP Leverage. We compute the public benchmark of the leverage ratio (debt to TEV ratio) for each industry and year pair (Market Leverage Ratio[Industry = N, Year = X]) as the average leverage ratio of Industry N in Year X. For each portfolio company, the Market Leverage component is computed as how much additional equity appreciation, comparing to the all-equity scenario, is generated by using debt that yields the same leverage ratio as the industry benchmark at acquisition. The rest of the Leverage component is attributed to GP Leverage component.

The last value creation component in the DIR framework is the Deleveraging component, which measures the impact of the change in net debt on equity appreciation during the GP's ownership. It is calculated as the ratio of the debt paydown (e.g., Net Debt at entry – Net Debt at exit) to TEV at entry. In total, there are six value creation components in the DIR framework, which sum to the equity growth during the holding period. A sample calculation is given in Table A1.

Table A1: Sample Calculation

Panel A: Inputs (in USD millions)

No.	Variable	Entry	Exit	No.	Variable	Entry	Exit
(01)	Revenue	\$422.7	\$513.3	**(06)**	TEV	\$723.3	\$1225.0
(02)	EBITDA	\$90.3	\$118.9	**(07)**	Net Debt	\$531.7	\$677.9
(03)	EBITDA Margin	21.3%	23.2%	**(08)**	Equity	\$191.6	\$547.1
(04)	EBITDA Multiple	8.0x	10.3x	**(09)**	Market Leverage Ratio	50%	
(05)	Market Multiple	7.5x	8.9x				

Panel B: Derived Variables

No.	Variable	Value	Calculation	No.	Variable	Value	Calculation
(10)	Revenue Growth	0.214	$\frac{(01)\ exit}{(01)\ entry} - 1$	**(11)**	EBITDA Growth	0.317	$\frac{(02)\ exit}{(02)\ entry} - 1$
(12)	EBITDA Margin Expansion	0.084	$\frac{(03)\ exit}{(03)\ entry} - 1$	**(13)**	EBITDA Multiple Expansion	0.286	$\frac{(04)\ exit}{(04)\ entry} - 1$
(14)	Market Multiple Expansion	0.181	$\frac{(05)\ exit}{(05)\ entry} - 1$	**(15)**	TEV Growth	0.694	$\frac{(06) exit}{(06) entry} - 1$
(16)	Combo[EBITDA, EBITDA Multiple]	0.091	$(11) \times (13)$	**(17)**	Combo[Revenue, EBITDA Margin]	0.018	$(10) \times (12)$
(18)	Combo[EBITDA, Market Multiple]	0.057	$(11) \times (14)$	**(19)**	Combo[EBITDA, GP Multiple]	0.034	$(11) \times [(13) - (14)]$
(20)	Market Debt at entry	361.7	$(06) \times (09)$	**(21)**	Equity Growth with Market Debt	1.387	$\frac{(06)\ exit - (20)}{(06) entry - (20)} - 1$
(22)	Equity Growth with Actual Debt	2.618	$\frac{(06)\ exit - (07) entry}{(06) entry - (07) entry} - 1$	**(23)**	Debt Paydown	-146.2	$(07)\ entry - (07) exit$

Panel C: Value Creation Components

Value Creation Components	Value	Calculation	%Equity Growth
Revenue Component	0.246	$(10) + 0.5 \times (17) + 0.25 \times (16)$	13.3%
EBITDA Margin Component	0.116	$(12) + 0.5 \times (17) + 0.25 \times (16)$	6.3%
Market Multiple Component	0.209	$(14) + 0.5 \times (18)$	11.3%
GP Multiple Component	0.122	$(13) - (14) + 0.5 \times (19)$	6.6%
Market Leverage Component	0.491	$(21) - (15) + (09) \times \frac{(23)}{(06)\ entry - (20)}$	26.5%
GP Leverage Component	0.873	$(22) - (21) + \frac{(23) \times [(07)\ entry - (20)]}{(08)\ entry \times [(06) entry - (20)]}$	47.0%
Deleveraging Component	-0.202	$(23) / (06)\ entry$	-10.9%
Equity Growth [MOIC -1]	1.855	$(08) exit / (08)\ entry - 1$	100%