Optimal Hedge Fund Allocation

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Draft version: July 15, 2025

Preliminary Draft: Comments Welcome

Abstract

This study addresses the optimal asset allocation problem for investors managing a diversified portfolio of stocks, bonds, and hedge funds. Significant allocations to hedge funds may be justified due to their diversification benefits, even when hedge funds generate minimal or no alpha. For instance, an investor with constant relative risk aversion and concern for inter-temporal utility should allocate around 20% to hedge funds, even under the assumption of zero alpha. Contrary to conventional wisdom, historical correlations and specified alpha levels indicate that equity and event-driven hedge fund strategies offer the greatest diversification advantages, while global macro and managed futures strategies are less favorable. However, optimal hedge fund allocations are highly sensitive to alpha assumptions. If alphas fall below -1%, the allocation to hedge funds typically approaches zero, whereas an alpha above 2% can lead the investor to allocate nearly 100% to hedge funds. This sensitivity also applies to individual hedge fund strategies. Finally, given that investing in many different hedge funds can be cost-prohibitive, we assess the allocation impact of investing in a limited number of hedge funds instead of a broad, uninvestable index. While reducing the number of hedge funds in a portfolio can substantially increase the likelihood that hedge funds will diminish investor utility when drawn from standard databases, we find compelling risk-adjusted performance when building allocations based on institutional-quality funds that are underrepresented in standard databases.

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I. Introduction

Allocations to alternative assets in institutional investment portfolios continue to increase. Studies suggest that the perceived historical outperformance and diversification benefits have driven the shift towards alternatives (see, for example, Goetzmann et al. (2019), Lerner et al. (2008), Gilbert and Hrdlicka (2015), and Binfarè et al. (2022)). In addition, Hochberg and Rauh (2012) find that many institutional investors are structured to benefit from the illiquid nature of most alternative investments. Yet, despite the growth in alternative investments, few careful empirical studies have examined their role in the overall risk-return profile of portfolios, particularly how an institutional investor would optimize their asset allocation when including alternatives. Data scarcity, short track records, illiquidity (e.g., lack of market prices for most alternative investment funds), and a dearth of high-quality index products all contribute to the difficulty of assessing optimal portfolio allocation.

Nonetheless, the literature has provided some limited understanding of the portfolio properties of certain alternatives. Brown et al. (2021b) find that adding private equity and private real estate funds to the traditional 60/40 portfolio of stocks and bonds improves the Sharpe ratio and Giommetti and Sorensen (2021) provide a theoretical model for optimal PE allocations. Jackwerth and Slavutskaya (2016) find that including hedge funds during times of financial crisis improved returns while reducing skewness and downside risk. However, Ang et al. (2014) find that investors would be willing to give up some of the return to limit exposure to illiquidity risk posed by alternative assets, and Brandon and Wang (2013) find that liquidity risk exposure has a significant impact on the performance of some hedge fund portfolios.

In this analysis, we focus on hedge funds and study the portfolio optimization problem a utility-maximizing investor faces. We examine 30 years of hedge fund data from a combination of commercial databases including BarlcayHedge, Bloomberg, EurekaHedge, HFR, and Morningstar as well as a novel dataset from PivotalPath, which provides coverage of many institutional-quality hedge funds unaccounted for in commercial datasets (see Barth et al. (2023) and Brown et al. (2025)). Our framework assumes that the risk relations across assets correspond to historical characteristics but that excess returns are forward-looking, so we can examine the optimal allocation under different scenarios. This allows us to examine the potential benefits provided by hedge funds, both overall and by sub-strategy. Specifically, we assume that our investor sets portfolio weights while optimizing over three assets (stocks, bonds, and hedge funds). When first assuming that hedge funds are not an option, we set expected returns so that the optimal allocation is 60% to stocks and 40% to bonds.[1] We use this implied 60/40 portfolio time series to create "zero-alpha" hedge fund returns while maintaining the risk profile of the respective time series. We can then create a set of return time series for hedge funds by adding incremental levels of alpha back to the zero-alpha returns. Of course, the investor's risk preferences play a significant role in the optimal allocation, and this poses an additional challenge for determining optimal portfolios with hedge funds. Specifically, some hedge funds suffer from large exposures to the downside, or "tail", risk (Agarwal and Naik, 2004) suggesting that the traditional mean-variance optimization may not be appropriate for portfolios with hedge funds. Bollen (2013) find that hedge fund investors are exposed to tail risk. Agarwal et al. (2017) show that hedge fund returns have systematic tail-risk exposure, and as such, this exposure is partly responsible for a portion of the variation in the cross-section of returns. Similarly, Gerakos et al. (2021) find that optimizations that maximized Sharpe ratios produced lower Sharpe ratios than optimizations focused on risk-factor exposures.

To better assess the trade-off between risk and return in portfolios with hedge funds, we determine the optimal allocations in two primary ways. We first take a traditional approach using mean-variance optimization (Kroll et al., 1984). We determine that funny things happen when the investor maximizes the Sharpe Ratio (i.e., mean-variance maximizing). We find that the allocation given to hedge funds is highly sensitive to the alpha level, and this sensitivity varies across fund strategies. This result holds when we expand the investment consideration set to 8 assets, including stocks, bonds, and all the hedge fund sub-strategies. In this setting, the combined total allocation to hedge funds jumps from 0% to over 90% when moving from 0% to 1% of annualized alpha.

Second, and as our main analysis, we assume the investor has constant relative risk-aversion (CRRA) preferences over the wealth of the portfolio at a terminal date T and at each intermediate period (month). This approach allows us to properly consider the impact of the return series' higher moments (skewness and kurtosis) in the investor's optimal allocation. When moving from mean-variance preferences to CRRA utility, we find a shift in which strategies are preferred. While the CRRA preference does penalize higher exposures to tail risk, which

[1] We do this by solving for the implied bond return for a given Sharpe ratio for which the 60/40 assumption holds. We then scale the time series of realized bond returns over the sample to have this implied expected return.

delays the allocation in some strategies, hedge funds quickly dominate the allocation as alphas increase. Again, the change in allocation is highly sensitive to the alpha level in the returns.

While evaluating the allocation question regarding hedge fund indices provides an important baseline context for understanding optimal allocations, the problem facing investors in the real world is even more complex. Specifically, investors can not buy a hedge fund index or a portfolio that holds most funds in an index. In practice, investors construct hedge fund portfolios with a finite number of funds (typically less than 50). To examine the effect of finite portfolio size on optimal allocations, we create portfolios by randomly selecting N individual hedge funds per month, create portfolios that consist of these funds' monthly returns, and simulate these portfolios 1,000 times. These simulated portfolios generate average annualized alphas around 1.2% during the full sample period. As expected, the alpha volatility is reduced when the number of funds selected per month increases. When creating portfolios of 50% stocks, 30% bonds, and 20% hedge funds, we find that including hedge funds improves the Sharpe ratio on average over the full sample period, compared to a portfolio of 60% stocks and 40% bonds. However, the investor may have lower CRRA utility, and the utility distribution has negative skewness, exposing the investor to greater left-tail outcomes. An investor who tries to gain exposure to hedge funds through fund-of-funds only exacerbates the potential negative outcomes, with lower performance and a greater probability of reducing the Sharpe ratio and CRRA utility.

Incremental to this analysis is a deeper (and updated) understanding the performance of the hedge fund industry. Evidence points to a decline of hedge fund performance over time (Dichev and Yu, 2011; Bali et al., 2013; Sullivan, 2021; Bollen et al., 2021). We add to these findings. We analyze the risk and return characteristics relative to global stocks, global bonds, and commodities as well as a large set of risk factors (SMB, HML, momentum, short-term reversal, long-term reversal, illiquidity, BAB, DVL, and QMJ).[2] When controlling for these factors in our regression analysis of hedge fund returns, we find that hedge funds have outperformed on average over the full sample period 1994-2023 across all strategies. However, these alphas only are observed in the first and second of three 10-year sub-periods we examine (1994-2003 and 2004-2013). Despite relatively good performance in 2020 and 2022, hedge fund alphas were statistically zero in the 2014-2023 sub-period. Given the central role hedge fund alphas play in the determination of optimal asset allocations, this deterioration in average industry

[2] This set of factors is representative of the span of return characteristics of hedge funds and is similar to those used in recent studies, including Brown et al. (2021a).

performance is consequential.

However, while the overall hedge fund industry has experienced a deterioration in *average* performance over time, as evidenced by declining average alphas and increasing correlation with traditional asset classes, it is important to recognize that this is not the full picture. Recent research, such as Barth et al. (2023) and Brown et al. (2025)), underscores that institutional-quality hedge funds continue to offer strong performance. In particular, institutional-quality funds are surprisingly underrepresented in commercial databases and their associated indices. However, they deliver higher alphas and lower correlations compared to the broader hedge fund universe, as captured in the standard commercial databases. For instance, funds analyzed using alternative data sources like Form PF and PivotalPath exhibit average alphas of two hundred basis points or more larger than those found in standard databases. These funds also have lower exposures to systematic risks, making them valuable components in a diversified portfolio. Such characteristics suggest that, while average hedge fund performance may be declining, our understanding of the hedge fund universe is limited by poor data. Further, institutional-quality hedge funds may instead provide substantial diversification benefits and robust risk-adjusted returns.

The remainder of the paper continues as follows: section II describes the data we use for our analysis, section III documents our findings on hedge fund performance, section IV discusses the implications for hedge fund allocations in diversified portfolios, section V analyzes fund-level implementation, and section VI concludes.

II. Data

Our sample of hedge funds consists of monthly return data between January 1994 and June 2023 from the combined dataset of BarlcayHedge, Bloomberg, EurekaHedge, HFM, HFR, PivotalPath, Morningstar, and ThomsonReuters (TASS). The data from PivotalPath provides better coverage of funds previously unaccounted for by traditional commercial datasets.[3] We believe our sample of hedge funds is the most complete and current dataset used in academic research to date.

[3] Barth et al. (2023) suggest that the size of the hedge fund industry is underrepresented in public databases due to the nature of which funds actively report and those that do not report at all. Brown et al. (2025) show that PivotalPath includes funds from many institutional-quality managers that do not report to commercial databases.

We first construct monthly return indices by hedge fund strategy as specified in our datasets. Because strategy taxonomies vary across data providers, we focus on the following seven strategies (which often combine sub-strategies): Credit, Equity, Event Driven, Global Macro, Managed Futures, Multi-strategy, and "All" which represents a composite of the six other strategies. We create monthly returns indices by taking an equal-weight average of all available monthly index returns from the various providers. Our index data may be more representative than it would have been if we had built it using only individual fund returns available in commercial databases since some data vendors (e.g., EurekaHedge) include "non-reporting" funds' returns in their indexes. To provide a representative picture to investors and industry observers, the non-reporting funds allow data vendors to use their returns to create indices but do not allow their fund-level returns to be included in databases.

Our fund-level implementation analysis gathers monthly returns for individual hedge funds from January 1994 through June 2023. We merge databases using the matching procedure outlined in the Data Appendix of Joenväärä et al. (2021). To eliminate duplicate share classes, we are careful to select a "representative" share class, such as the one with the longest return series and the largest assets. Instead, we aggregate the fund-level information across all duplicates and create a "master" share class using information across databases and different share classes. Once combined, our initial dataset contains over 3 million fund-month observations.

Our data is otherwise cleaned in the following ways. We winsorize returns at the 99% level. Fund managers may only start reporting to commercial databases once they have a good track record of returns. This behavior leads to a backfill bias in the initial periods of the reporting funds returns. To account for this bias, we remove the reported monthly returns for each fund before the respective database's recorded listing date, following a common practice in the literature (Bhardwaj et al., 2014). We impute missing listing dates using the Jorion and Schwarz (2019) method and use the earliest possible listing date for funds reporting to multiple databases (Joenväärä et al., 2021). We restrict our sample to funds that maintain continuous monthly reporting to overcome any bias arising from funds that choose to take breaks in reporting. We require that funds have at least 12 fund-month observations. We then manually create a standardizing matching process of main strategy categories for each fund to the overall strategy categories we use for our index-level analysis. We keep only those funds that appropriately match one of these main strategy categories. We separate fund-of-funds (392,075 fund-month observations for 5,372 funds), which we examine separately. Our final sample of primary funds is an unbalanced panel of 20,222 individual funds with 1.35 million fund-month observations.

We collect market data from a variety of sources. We utilize Bloomberg to collect data on global stocks, global bonds, commodities, U.S. stocks, and U.S. bonds. We use the MSCI World Total Return Index, Bloomberg Barclays Global Aggregate Total Return Index, and the S&P GSCI Index for global stocks, bonds, and commodities, respectively. For U.S. market exposures, we use the Vanguard Total Stock Market Index Fund for stocks and the Bloomberg U.S. Aggregate Total Return Value Index for bonds. We utilize a set of risk factors similar to those used in recent studies, such as Brown et al. (2021a). These factors include exposures to Small Stocks (SMB), Value Stocks (HML), Momentum, Short-term Reversal, and Long-term Reversal from the Ken French Data Library. We use the Pástor and Stambaugh (2003) traded liquidity risk factor to account for exposures to the changes in liquidity and source this data from Robert Stambaugh's website. We also include Betting against Beta (BAB), The Devil's in HML's Details (DVL), and Quality minus Junk (QMJ) obtained from the AQR website (see Frazzini and Pedersen (2014), Asness and Frazzini (2013), and Asness et al. (2019)). While many other factors have been proposed in the literature, our analysis suggests that these nine risk factors and the market factors span the vast majority of common risks in our hedge fund dataset.

Given the nature of how hedge fund returns are calculated and reported, our return data are potentially subject to serial correlation, which creates a smoothing bias. We unsmooth the index returns from the commercial datasets using a moving average (MA) model following Getmansky et al. (2004). Testing the MA model parameters, we determine that an MA(1) model performs sufficiently well for our analysis. Autocorrelation is present in the first lagged period for most hedge fund strategy indices. While autocorrelation extends into two or more lagged periods, moving to an MA(2) model does not improve performance over the MA(1) in our setting or modify our results. We then take the unsmoothed commercial dataset hedge fund returns and create our composite strategy indices. This gives us an unsmoothed time series of monthly returns for each hedge fund strategy over the sample time frame of February 1994 through June 2023. Unsmoothing the composite indices provides very similar results.

We report summary return statistics of raw and unsmoothed hedge fund returns and the market and risk factors in Table I. Unsmoothing hedge fund returns removes much of the autocorrelation, reducing these values to levels similar to the market returns and risk factors. Unsmoothing should leave average returns about the same but increase the variance of returns.

We find that after unsmoothing, the mean return only decreases by 0.3% while the standard deviation increases by about 15% (for example, from 5.9% to 6.8% for the *All* index). Following the same procedure, we also unsmooth the fund-level returns of our sample.

III. Hedge Fund Performance

A. Hedge Fund Alphas and Factor Exposures over The Full Sample

To begin our analysis of hedge fund performance, we risk-adjust hedge fund returns by controlling for their exposures to risk factors, e.g., Fung and Hsieh (2001) and Agarwal and Naik (2004). We take a similar approach by utilizing a set of risk factors that incorporates those used in the literature, including a few relatively recent factors. This group of factors provides a satisfactory span of the risk exposures faced in the hedge fund industry. It allows for robust inference without relying on a larger set of factors or a specific model selection technique.

We estimate alphas from the regression of the fund index monthly returns (in excess of the one-month Treasury bill rate) on the monthly excess returns of three market factors (global stocks, global bonds, and commodities) and the monthly time series of nine risk factors (SMB, HML, momentum, short-term reversal, long-term reversal, illiquidity, BAB, DVL, and QMJ) for the full sample, January 1994 through June 2023. Table II reports the results from this regression. We find that hedge funds, in aggregate, have tended to have positive risk-adjusted returns over the last three decades, with all strategies except *Managed Futures*, having alphas significantly greater than zero at the 95% or better confidence level. The annualized alphas range from 3.5% for *Managed Futures* to 4.4% for *Event Driven*. Taken at face value, these levels of alpha suggest a return-maximizing investor should allocate a large portion of their portfolio to hedge funds.

Several commonalities between the different strategies emerge when considering the factor exposures in the regression results of Table II. The *All*, *Credit*, *Equity*, *Event Driven*, and *Multi-strategy* indices tend to have similar significant factor loadings. These strategies have statistically positive betas, most at the 99% level, for the market factors *Global Stocks* and *Commodities*, as well as the risk factors *SMB*, *BAB* (except for *Equity*), and *QMJ*. The similarities suggest that these strategies primarily implement trades that create exposure to global stock and commodity markets, exhibiting characteristics of relatively long small stocks with low market betas that have low "quality" scores across profitability, growth, safety (low idiosyncratic volatility, etc.). In addition, the magnitude of exposures is relatively similar as well. While there are differences between the full set of strategies, we generally find that hedge fund strategies demonstrate fairly common risk attributes that may limit the benefits of diversifying across strategies.

Next, we examine the alphas and factor exposure changes over the full sample by estimating rolling regressions over 36-month windows. Patton and Ramadorai (2013) find that exposures to risk factors vary significantly over time, even across and within months. Regarding understanding the return performance of hedge funds, Bollen and Whaley (2009) suggests that time-varying risk factors can impact analysis results when studying risk-adjusted alphas. In Figure 1 we show the plots of the rolling alphas and betas of global stocks and global bonds for the hedge fund strategy indices. As was suggested earlier, we find that the 36-month rolling alpha of the average hedge fund has steadily declined since the 1990s to levels close to zero in 2023. We also find that the risk exposures of the hedge funds in aggregate change dramatically over the sample, with periods of convergence in exposure between the different strategies.

B. Hedge Fund Returns and Alphas over Sub-periods

We also examine the performance of hedge funds and document the change in return characteristics over our sample period. This is useful when we subsequently examine how optimal allocations vary through time. Here we consider performance over three 10-year sub-periods: 1994-2003, 2004-2013, and 2014-2023. We repeat the regression analysis used on the full sample of monthly returns for each of the sub-periods and report the regression outputs for the Hedge Fund Strategy *All* index in Table III. We find statistically and economically significant alphas for the *All* index in the first two sub-periods. The middle sub-period, 2004-2013, exhibits a 56% decrease in alpha and shows a shift in the beta exposures from *Global Stocks* and *SMB* to *Quality minus Junk* with a more negative loading. In the most recent sub-period, we find alphas statistically indistinguishable from zero and showing a statistically significant positive loading on *Illiquidity* for the first time, suggesting a fundamental aggregate shift in hedge fund strategies.

In Table IV, we first report the annualized excess return, annualized volatility, and Sharpe ratio for each of our hedge fund strategy indices over full sample and sub-periods. Average hedge fund returns have decreased over the last 30 years across all strategies. Bollen et al. (2021) find that hedge fund returns declined between 2008-2016 and document a similar trend, namely a decline in returns across all strategies between 2004-2013 and 2014-2023. Note that considering only returns can obscure the risk-adjusted performance. Likewise, the Sharpe ratios demonstrate a downward trend for all strategy indices since the 1990s.

Table IV also shows the fairly stable volatility of the hedge fund strategy indices. We find that the average volatility of these monthly returns typically ranges between 5%-8% for the different sub-periods. Given this volatility dynamic, we find that changes in annualized returns primarily drive changes in the Sharpe ratios. The sensitivity of the Sharpe ratios to the level of hedge fund returns gives us our first inclination of how sensitive the optimal allocation of hedge funds is to the alpha level in the returns.

Next, we focus on three regression outcome variables: the annualized alpha, information ratio, and R-squared. These are useful in clarifying how hedge funds have performed over the sub-periods. The annualized alphas characterize performance after controlling for a relatively extensive set of relevant factors. The information ratio (calculated as the regression alpha divided by the standard deviation of the regression residual) provides insight into the comparison between the level of outperformance weighted by a measure of tracking error. The R-squareds measure the uniqueness of hedge fund strategy risk.

We isolate these variables of interest from the full sample regression and sub-period regressions and summarize the findings in Table IV as well. Over the full sample of 1994-2023, the hedge fund strategy indices all have statistically significant alphas, most at the 99% level, with *Managed Futures* significant at the 95% level. These significance levels generally hold only when evaluating the first sub-period, 1994-2003, with *Managed Futures* significant at the 90% level. In the 2004-2013 sub-period, alphas mostly remain statistically greater than zero but are always smaller, typically about half the size of those in the 1994-2003 sub-period. However, we find that annualized alphas for the average hedge fund are statistically and economically close to zero during 2014-2023. The decline of alpha over the three sub-periods is robust to other risk models, including using only *Global Stocks* and *Global Bonds* as factors.

The information ratios over the sub-periods reveal a somewhat different pattern in performance over time by strategy. Notably, the decline in the information ratios is not as drastic between the 1994-2003 and 2004-2013 sub-periods for most of the strategies. In particular, *Credit* and *Managed Futures* barely decline. On average, the information ratios only decreased by about a quarter between the first two sub-periods. However, in the 2014-2023 sub-period, we found a decline in the information ratios to nearly zero.

Examining the R-squared's from the factor regressions reveals an increase over the three sub-periods for most of the strategies. And except for *Global Macro* and *Managed Futures*, by the 2014-2023 sub-period, nearly 90% of the variation in the average hedge fund's returns is explained by variations in the returns of our set of market and risk factors. R-squared's for *Global Macro* and *Managed Futures* are both lower and more stable over time. These findings suggest that not only are alphas declining to close to zero over our sample, but that hedge fund returns are increasingly likely to be driven by common factor exposures. Prior studies compare high and low performing funds, such as by Kosowski et al. (2007). Our analysis shows that if an investor could reasonably diversify across all funds within a given industry over the sub-periods, this investor's portfolio would experience a reduction in expected alpha and be increasingly correlated with market and risk factor returns.

IV. Hedge Fund Allocation

As documented in the previous section, alphas have disappeared across all strategies for the average hedge fund in recent years, as represented by our hedge fund strategy indices. We do our main regressions on market factors (global stocks, global bonds, and commodities) and risk factors (SMB, HML, momentum, short-term reversal, long-term reversal, illiquidity, BAB, DVL, and QMJ). While it is important to have a set of extensive relevant factors to avoid overstating any significant alpha, our result that alphas are statistically indistinguishable in aggregate from zero in the last sub-period, 2014-2023, remains even when only controlling for two market factors, *Global Stocks* and *Global Bonds* or for *US Stocks* and *US Bonds*. This finding leads us to question how the decline or disappearance of hedge fund alpha impacts the allocation given to hedge funds in a diversified portfolio. We fully acknowledge that individual hedge funds in the cross-section could remain valuable, even in the most recent sub-period. However, access to top-performing hedge funds may be limited, similar to Binfarè et al. (2022). As such, we focus our analysis below on the average institutional investor's portfolio allocation to the average hedge fund. We build on this foundational understanding when we discuss the feasibility of implementation later.

A. *Hedge Funds Added to the 60/40 Stocks/Bonds Portfolio*

Prior academic works suggest the inclusion of hedge funds in portfolios of stocks and bonds can provide added benefits to investors (see Bollen et al. (2021) and Jackwerth and Slavutskaya (2016)). In our initial examination of the potential benefits gained by adding hedge funds to the traditional 60/40 portfolio of stocks and bonds, we first create a time series of monthly returns that are a weighted average of 60% *US Stocks* and 40% *US Bonds* over the full data sample, January 1994 through June 2023. Keeping the relative 60/40 ratio between stocks and bonds constant, we then create monthly returns of a portfolio that allocates a portion to the Hedge Fund Strategy *All* index, in values of 10%, 20%, 30%, and 40%. We then calculate the average return, volatility, Sharpe ratio, skewness, and kurtosis of each respective time series of monthly returns.

We report the results in Table V, both for the full sample period and for each of the sub-periods: 1994-2003, 2004-2013, and 2014-2023. The 0% column represents the baseline 60/40 portfolio, consisting of only stocks and bonds. Each subsequent column shows the analysis results for the corresponding percent allocation to hedge funds. When considering the full sample, we find allocating a higher portion of the portfolio to a well-diversified set of hedge funds, represented here by the Hedge Fund Strategy *All* index, provides investors with increased annualized returns and a reduction in the annualized volatility. These changes exemplify the upward-sloping shift in the Sharpe ratio from 0.50 in the base 60/40 portfolio to 0.67 when hedge funds have a 40% allocation. This result agrees with the finding of Brown et al. (2021b), as we also find that replacing a portion of the allocation that was given to traditional assets with an alternative asset, namely, hedge funds in our case, improves the Sharpe ratio of the investor's portfolio. While adding hedge funds benefits average returns and volatility, this also creates more negative skewness, corresponding to a longer left-tail in the distribution of returns.

Additionally, increasing the allocation to hedge funds over the full sample increases the portfolio's kurtosis, i.e., the distribution of returns has larger tails. Our analysis suggests a mean-variance maximizing investor will choose an allocation as large as possible to hedge funds. However, in agreement with Agarwal and Naik (2004), the additional allocation brings potential negative consequences that would be missed in this traditional maximization framework.

Examining the results for the sub-periods provides initial insight into the diversification benefits of hedge funds to the 60/40 stocks/bonds portfolio. In the first sub-period, 1994-2003, we find a similar pattern of costs and benefits to that of the full sample. In this earliest time frame, hedge funds receiving a 40% allocation equates to a Sharpe ratio improvement of 78.7%, increasing from 0.43 to 0.77. The skewness of the portfolio also becomes marginally more negative while the kurtosis increases to 0.70, giving rise to fatter tails with more left-tail events.

The middle sub-period provides a slightly different view of an allocation to hedge funds. In 2004-2013, the average return increased by 10.6%, and the volatility decreased by 15.6%, which resulted in a 30.9% improvement of the portfolio Sharpe ratio, moving from 0.48 to 0.63. While the improvement to the Sharpe ratio in this middle sub-period is less than in the first sub-period, a distinctive difference comes from the impact on the skewness and kurtosis. The skewness of the portfolio remains relatively constant across all levels of hedge fund allocation, and the kurtosis declines by 14.8%. This suggests that during this time frame, the inclusion of hedge funds in diversified portfolios was a net benefit, agreeing with the findings of Jackwerth and Slavutskaya (2016), by improving the Sharpe ratio and reducing skewness and kurtosis.

We observe a very different situation in the last sub-periods, 2014-2023. Giving a higher allocation to hedge funds reduces the portfolio's average return while decreasing the volatility. These two combined effects still result in an overall increase to the portfolio Sharpe ratio but only by 5.8%, a change of 0.57 to 0.60 when moving the hedge fund allocation from 0% to 40%. While the Sharpe Ratio improves, the skewness becomes more negative with the largest relative change of any sub-periods, 16.6%. Additionally, the kurtosis more than doubles when considering the shift from a 0% to 40% hedge fund allocation in the last sub-period. From this analysis, we find the inclusion of hedge funds into a diversified portfolio can be seen as beneficial over the full sample. Yet, those benefits and costs can reverse depending on the performance characteristics of the underlying stocks and bonds of the 60/40 portfolio.

B. Mean-Variance Optimization with Increasing Alphas

The prior approach of creating portfolios between the 60/40 stocks/bonds portfolio and increasing allocations to the average hedge fund gives us a first-pass view of the potential benefits and costs of adding hedge funds to diversified portfolios. To build on these initial insights, we take a more complex approach by assuming that a representative agent, the portfolio manager (PM), wants to maximize their utility and that the PM's utility is directly tied to the performance of the portfolio, a combination of stocks, bonds, and hedge funds. Kroll et al. (1984) suggest that mean-variance maximization can serve as a proxy for direct utility maximization, and as such, we assume the PM wishes to select asset allocation weights that maximize the portfolio Sharpe ratio,

$$\max_{W} \frac{W'\bar{r}}{(W'\Sigma W)^{\frac{1}{2}}}, \tag{1}$$

where W is the vector of portfolio weights, $\bar{r}$ is the vector of asset returns, and Σ is the covariance matrix of realized returns.

We seek to understand how changes to the level of alpha impact the hedge fund allocation given by an optimizing PM. We start by removing the alpha from the hedge fund returns to answer this question. We use these zero-alpha returns to examine if utility or Sharpe ratio maximizing PMs still wish to allocate hedge funds in a setting of no alpha. To calculate our zero-alpha hedge fund returns, we run the following regression:

$$r_{hf,t} = \alpha_{hf} + \beta_{60/40} \times r_{60/40,t} + \epsilon_{i,t}, \tag{2}$$

where $r_{hf,t}$ are the monthly returns for hedge fund strategy index hf and $r_{60/40,t}$ are monthly returns of the 60/40 (stocks/bonds) portfolio. We then take the $\beta_{60/40}$ from the regression to calculate the implied zero-alpha expected return:

$$\bar{r}^{imp}_{hf} = \beta_{60/40} \times \bar{r}_{60/40}, \tag{3}$$

where $\bar{r}_{60/40}$ is the average monthly return of the 60/40 portfolio. We then create the series of implied monthly zero-alpha hedge fund strategy index returns by scaling the realized hedge fund strategy index returns by the implied average hedge fund strategy index return:

$$r^{imp}_{hf,t} = r_{hf,t} - \bar{r}^{imp}_{hf}. \tag{4}$$

As suggested by Giommetti and Sorensen (2021), adding alternative assets into an optimizing agent's consideration set will modify the baseline allocations of the agent. We assume that the PM will choose an allocation of 60% stocks and 40% bonds when hedge funds receive no allocation. To ensure this base assumption is satisfied, we utilize the Sharpe ratio formula, and we solve for the implied average bond return given the Sharpe ratio such that a mean-variance optimizing PM requires choosing the 60/40 portfolio when optimizing over three assets: stocks, bonds, and hedge funds,

$$SR_{60/40} = \frac{W'\bar{r}}{(W'\Sigma W)^{\frac{1}{2}}} = \frac{w_s\bar{r}_s + w_b\bar{r}_b}{(W'\Sigma W)^{\frac{1}{2}}}, \tag{5}$$

where $SR_{60/40}$ is the Sharpe ratio of the 60/40/0 portfolio, w_i is the portfolio weight of asset i, and $\bar{r}_i$ is the average return of asset i. We rearrange the above equation to solve for the implied average bond return, which gives

$$\bar{r}_b^{imp} = \frac{SR \times (W'\Sigma W)^{\frac{1}{2}} - w_s \times \bar{r}_s}{w_b}. \tag{6}$$

We then create the series of implied monthly bond returns by scaling the realized bond returns by the implied average bond return:

$$r_{b,t}^{imp} = r_{b,t} - \bar{r}_b^{imp}. \tag{7}$$

Next, we form the implied monthly 60/40 returns:

$$r_{60/40,t}^{imp} = 0.6(r_{s,t}) + 0.4(r_{b,t}^{imp}). \tag{8}$$

We use this $r_{60/40,t}^{imp}$ in the regression process previously discussed in equations (2), (3), and (4) to create the implied zero-alpha hedge fund returns, $r_{hf,t}^{imp}$. Importantly, this keeps the covariance matrix, Σ, the same, only modifying the level of returns. We then analyze the allocations made by a Sharpe ratio maximizing PM of a portfolio formed from r_s, r_b^{imp}, and r_{hf}^{imp} and solve for the optimal portfolio weights. In this setting, each hedge fund strategy index achieves zero alpha above the 60/40 portfolio based on the realized return of stocks and the implied return of bonds. For the full sample period, we find that a Sharpe ratio of 0.425, implying an annualized bond excess return of 0.44% and an annualized 60/40 portfolio excess return of 4.19%, is sufficient to entice the PM to choose the 60/40 portfolio when optimizing over three assets, stocks, bonds, and hedge funds, when hedge funds receive no allocation. Next, we add incremental levels of alpha to the hedge fund strategy indices and rerun the optimization. From this, we can find the optimal portfolio weights for each level of added-back alpha while keeping the covariance structure, Σ, constant between the three assets. Therefore, we can test how the level of alpha impacts the allocation choice of the PM who is maximizing the portfolio's Sharpe ratio.

In Figure 2, we report the allocation to the hedge fund strategy indices at each level of added-back alpha for a set of annualized alphas between 0.0% and 4.0%. The point 0.0% on the x-axis represents where the average hedge fund achieves no alpha and, correspondingly, receives no allocation in the portfolio by the PM across all strategies. Given the PM's trade-off between the level of return and the volatility of the returns, while hedge funds provide lower volatility, the lack of alpha prevents the PM from giving any allocation to hedge funds. As we increase the level of added-back alpha, the optimal portfolio allocation is highly sensitive to the alpha level. Even a marginal alpha level change has a sizable impact on the hedge fund allocation. At an annualized added-back alpha level of 0.5%, hedge funds receive allocations ranging from nearly 15% for *Managed Futures* to over 40% for *All*, *Multi-strategy*, and *Credit*. These allocations quickly increase to 80%-100% for most strategies at annualized alpha levels of only 1.5%-2.0%.

Interestingly, the increased allocation to hedge funds decreases the overall portfolio return. However, this reduction in returns is offset by the greater decrease in portfolio volatility, and as such, the portfolio Sharpe ratio can remain the same or increase. Over the incremental increase of annualized alphas from 0.0% to 0.5%, the portfolios' Sharpe ratios show very marginal increases. Yet, the change in the allocation to hedge funds is substantial, creating meaningful changes in the portfolio return and volatility.

Figure 2 also provides supplemental charts to those already discussed. As the Sharpe ratio maximizing PM gives more allocation to hedge funds at increasing levels of alpha, the decrease in the allocation to stocks and bonds is not equal between the two assets, and the trade-off between stocks, bonds, and hedge funds is heterogeneous across strategies. For example, while *Credit* and *All* share a similar curvature in the changes in hedge fund allocation, at the annualized added-back alpha level of 1.0%, the optimal portfolio weights for *Credit* consist of approximately 15/3/82 for stocks/bonds/hedge funds and for *All* are approximately 9/19/72. Thus, the PM with *All* in their consideration set pulls away from stocks faster and maintains a higher proportion of bonds than if considering *Credit* instead, despite having the same starting point of 60/40 stocks/bonds. Indeed, there are large differences between the strategies in how the optimizing PM allocates to the three assets at increasing alpha levels. In the last two charts, we find no consistent pattern in the changes in skewness or kurtosis, which suggests that the PM does not consider the potential costs of these higher moments when making hedge fund allocations. This is evident as strategies that improve the skewness and kurtosis, such as *Equity*, receive allocations at a slower rate than those that create added risk exposure to these moments, such as *Credit* and *Multi-strategy*.

We next consider how the PM might choose optimal portfolio weights when given an expanded consideration set of stocks, bonds, and 6 of the hedge fund strategy indices. We exclude the Hedge Fund Strategy *All* index as this represents the diversification over all the other strategies, and we seek to understand the interplay between the strategies in our optimization. We repeat the same exercise performed in the 3-assets case with the full set of 8 assets for levels of annualized added-back alphas between 0.0%-4.0%. The findings are reported in Figure 3. When none of the hedge fund strategy indices have alpha, the optimizing PM maintains the base case 60/40 stocks/bonds portfolio. Similar to the results in the 3-asset case, the overall allocation to hedge funds is highly sensitive to the alpha level.

For the annualized added-back alpha of 1.0%, hedge funds collectively receive an allocation of around 91%, and the breakdown of allocations of the total portfolio is approximately 8/1/91 for stocks/bonds/hedge funds. Within the overall hedge fund allocation, *Credit* and *Multi-strategy* receive the highest weights of around 45% and 25%, respectively. *Futures* (around 11%), *Equity* (around 9%), and *Global Macro* (around 1%) are the other hedge fund strategy indices to receive an allocation. Also, similar to the 3-asset case, the optimizing PM is willing to sacrifice portfolio return to reduce portfolio volatility. Thus, the PM can increase the Sharpe ratio through the denominator while at the same time allowing the skewness to become more negative and to have higher kurtosis overall. Interestingly, given the slope of the decreasing allocations to stocks and bonds, it appears that when optimizing over the set of 8 assets, the PM pulls from stocks and bonds at an equal rate based on the curvature of their allocation paths, and the linear slope between the baseline starting allocations and the point of zero allocation.

C. CRRA Optimization with Increasing Alphas

When a PM performs mean-variance optimization, they neglect the consideration of increased risk from higher moments. To address this concern, we will assume the PM has CRRA utility preferences regarding portfolio wealth.

We assume the PM starts with \$1 of wealth at $t = 0$ and gains the portfolio return for a given month ($return_t$) over a paid dividend each month. The dividend (div) is the average monthly 60/40 stocks/bonds portfolio return over the respective sample period. This allows us to gauge how the PM's portfolio performs relative to the expected return of the traditional 60/40 portfolio. This is necessary to maintain the curvature of the CRRA function, which is vital in our optimization process.

Wealth at time t is given by $wealth_t = wealth_{t-1} \times (1 + return_t - div)$. The PM then seeks to maximize expected utility in three cases, as given in the following:

Preferences over the wealth of the portfolio at a terminal date T:

$$E[U(wealth)] = \begin{cases} \ln(wealth_T) & \text{for } \gamma = 1, \\ \frac{(wealth_T^{1-\gamma})-1}{1-\gamma} & \text{for } \gamma > 1 \end{cases} \tag{9}$$

Preferences over the wealth of the portfolio at each intermediate period (intertemporal):

$$E[U(wealth)] = \begin{cases} \frac{1}{T}\sum_{t=1}^{T} \ln(wealth_t) & \text{for } \gamma = 1, \\ \frac{1}{T}\sum_{t=1}^{T} \frac{(wealth_t^{1-\gamma})-1}{1-\gamma} & \text{for } \gamma > 1 \end{cases} \tag{10}$$

Preferences over a combination of the wealth of the portfolio at a terminal date T and at each intermediate period:

$$E[U(wealth)] = \begin{cases} \theta \times [\ln(wealth_T)] + (1-\theta) \times \left[\frac{1}{T}\sum_{t=1}^{T} \ln(wealth_t)\right] & \text{for } \gamma = 1, \\ \theta \times \left[\frac{(wealth_T^{1-\gamma})-1}{1-\gamma}\right] + (1-\theta) \times \left[\frac{1}{T}\sum_{t=1}^{T} \frac{(wealth_t^{1-\gamma})-1}{1-\gamma}\right] & \text{for } \gamma > 1 \end{cases} \tag{11}$$

where θ represents the relative weight of the first component (in our analyses below, we set it to 50%).

We utilize the same assumption satisfying the Sharpe ratio discovered in the Sharpe ratio maximizing case to determine the implied bond returns for a given Sharpe ratio that entices the PM to choose the 60/40 stocks/bonds portfolio when optimizing over stocks, bonds, and hedge funds, but when hedge funds receive no allocation. Recall that over the full sample period, a Sharpe ratio of 0.425 satisfies this assumption. This gives us an implied bond excess annual return of 0.44% and an excess annual return of 4.19% for the 60/40 portfolio. We follow the same procedure in creating zero-alpha hedge fund strategy indices and then allow the CRRA maximizing PM to pick the optimal weights for varying levels of added-back annualized alpha.

In Figure 4 we show the optimized allocation between 3 assets, stocks, bonds, and a hedge fund strategy index over different levels of risk aversion γ. Setting $\gamma = 5$, in all three CRRA utility cases, the baseline portfolio is the 100% stocks rather than the 60/40 stocks/bonds portfolio. The trade-off in these cases reflects only the choice between stocks and hedge funds, as bonds never receive an allocation. We find in the CRRA terminal utility case that the slope of the allocation curves for the hedge fund strategy indices is very steep and begins above annualized alphas of 2%, as the PM cares deeply about the absolute return of the portfolio and thus allocates to hedge funds only when the returns are high enough to justify the shift from stocks. When the PM cares about the intertemporal concerns of the portfolio or a combination with the terminal wealth, we find a lower slope of the allocation curves, with hedge funds receiving allocation even with negative alpha.

Considering a higher level of risk aversion, $\gamma = 15$, we find the PM still maintains a base portfolio of 100% stocks and does not allocate hedge funds until above annualized alphas of 2% in the CRRA Terminal Utility case. However, the allocation to hedge funds does not increase directly from 0% to 100%, but rather, the change exhibits curvature and slows around 55% for levels of reasonable alpha. We find that bonds receive an allocation for some CRRA Intertemporal Utility case observations. In addition, given CRRA preferences over a combination of terminal and intertemporal, we find the PM will approximately choose the 60/40 stocks/bonds portfolio when hedge funds receive no allocation. As the level of annualized alphas increases, hedge funds first replace bonds in the portfolio, as demonstrated by the relatively steep and straight uptick in allocation. The kink in the curve represents where the portfolio consists only of stocks and hedge funds. The allocation to hedge funds follows a slower, monotonically increasing curve until hedge funds completely replace stocks. Shifting γ from our base case of 5 to 15 demonstrates that the sensitivity to alpha declines in the allocation choice as the PM becomes more risk-averse.

To explore the optimal allocation choice more fully, we consider the PM with CRRA preferences over both terminal and intertemporal utility, and we set $\gamma = 15$ to recover a base case of the 60/40 stocks and bonds portfolio. In Figure 5, we report the results from the optimizations for a range of added-back annualized alphas of -2.0% to 4.0%. When the PM optimizes over CRRA utility, the implicit requirement of some strictly positive alpha for hedge funds to receive allocation no longer exists. In fact, at the point 0.0% of added-back annualized alphas, all strategies have a positive allocation, with *Equity* receiving over 60% and positive allocation beginning at added-back annualized alpha below -2.0%. While *Credit* and *Multi-strategy* were the front-runners for the mean-variance optimizations, these strategies do not receive any allocation until around 1.0% of annualized alpha. Across all strategies, the PM chooses to sacrifice both returns and volatility, thus moving to lower Sharpe ratios to improve the average CRRA utility over the wealth of the portfolio. The trade-off between stocks and bonds is fairly similar across strategies, as all strategies seem to immediately move away from bonds when the level of annualized hedge fund alpha is sufficient, dropping from the base case of 40% bonds to 0% bonds relatively quickly. In this scenario, the changes in the skewness and kurtosis impact the hedge fund allocation, favoring strategies that do not create more negative skewness or higher kurtosis. However, this impact is quickly dominated by the level of hedge fund alpha. We still find that the optimal allocation given to hedge funds remains very sensitive to the alpha level in those returns.

The allocation is so sensitive to the level of alpha that when we attempt to increase the consideration set of the PM to include eight assets (stocks, bonds, and the hedge fund strategy indices expect *All*), the PM chooses allocations that coincide with the strategy that receives higher allocations at lower levels of alpha. Thus, when we optimize over the eight assets, the output charts are identical to the 3-asset case when considering stocks, bonds, and *Equity*. When we remove the dominating strategy index, which in the base case is *Equity*, and rerun the optimization on the remaining set of hedge fund strategy indices with stocks and bonds, the portion of hedge fund allocation shifts to the next dominating strategy index, *Event Driven*, such that the output charts are the same as optimizing only over those three assets.

D. *Utility Optimization over Sub-periods*

We now compare the hedge fund allocations in the mean-variance and CRRA utility optimizations across sub-periods, reported in Figure 6 and Figure 7. In these allocation charts, we consider the slope of the allocation curves and the alpha level for the initial allocation in the CRRA case to indicate the diversification benefit of including the respective hedge fund strategy index in a portfolio of stocks and bonds. This can also be interpreted as the allocation's sensitivity to the alpha of hedge funds. We find the slopes of the allocation curves shift between sub-periods, suggesting that diversification provided by hedge funds depends on the sub-period.

For the mean-variance optimizing PM, hedge fund allocation relies on alpha during the middle sub-period, 2004-2013. This suggests that the average hedge fund provided the least amount of respective diversification benefit, in terms of the Sharpe ratio, during the Great Financial Crisis. While the allocation curves have similar slopes during 1994-2003 and 2014-2023, the ordering of the different hedge fund strategy indices changes. Over the full sample, *Credit*, *Multi-strategy*, and *All* are the first three to receive allocations as alpha increases, and this holds for 1994-2003 and 2004-2013. For the most recent sub-period, 2014-2023, this ordering changes such that the slope of the allocation curve for *Credit* is lower and *Equity* is higher, suggesting that the diversification provided by different strategies is also highly dependent on the sub-period.

We also find changes in the slopes of the hedge fund allocation curves and the alpha level for initial positive allocation when considering optimizing over CRRA utility. Hedge fund allocation slopes during the middle sub-period, 2004-2013, beginning at lower levels of alpha. Hedge funds start receiving allocations for all strategies, excluding *Credit* at levels of negative alpha. This suggests the average hedge fund may have provided diversification in terms of higher moments during extreme market downturns but potentially less during more sustained bull markets. In the other sub-periods, some positive alpha is required for the PM to allocate to the average hedge fund, and the slopes of the allocation curves are very steep, especially in 2014-2023, and require at least 3% of annualized alpha. Thus, to maximize the diversification benefit of hedge funds, a PM must update allocation rules to reflect the current market conditions rather than sticking to common heuristics and be cognizant of what measure of utility is desired.

V. Fund-Level Implementation

Evaluating the performance of alternative investments by its average performance does not account for the idiosyncratic risk exposure and is therefore overly optimistic (Gredil et al., 2021). Additionally, Joenväärä et al. (2019) find added investment constraints prevent investors from accessing the return performance found in prior studies. In regards to persistence, capital flow constraints create barriers for alpha producing funds to continue to outperform in the future as suggested by Fung et al. (2008). Also, Jagannathan et al. (2010) find persistence in superior funds but not for inferior funds. So far, we have only considered the allocations given to hedge funds in diversified portfolios based on the returns of our constructed hedge fund strategy indices. In practice, however, the feasibility of diversifying over the entire hedge fund space, or even one of the hedge fund strategies, is very unlikely. Hedge fund portfolios are finite and have restrictions on participation. Even the largest, most sophisticated managers typically invest in less than 50 funds, and investing through Fund-of-funds entails an additional layer of fees. The trade-off we wish to explore is the selection of individual funds, which may create significant improvements to a PM's portfolio, balanced against the feasibility and practicality of consistently picking the funds that do so, given the funds' idiosyncratic risk.

We modify our prior analysis of the utility-maximizing PM, replacing the hedge fund strategy indices with a realistically sized set of randomly selected hedge funds. Using the collection of 1.33 million fund-month observations of individual hedge funds between February 1994 and June 2023, we implement two selection strategies, randomly selecting $N = 5, 10, 30$ funds per month and assuming no reallocation frictions for the PM. We first set the selection to be purely random, and each month, the algorithm considers the full set of fund observations and selects N funds randomly to hold for that month. In the second approach, the funds are selected in such a way as to match criteria based on the strategy of the funds. For $N = 5$, the algorithm selects one fund at random for each of the equity, credit, global macro, event-driven, or futures strategy funds available for that month, giving us one fund of each strategy for each month. For $N = 10$ and $N = 30$, we select funds according to a strategy weighting matrix such that each month, our portfolio of funds will contain 30% equity, 20% credit, 20% global macro, 10% event-driven, 10% futures, and 10% multi-strategy in an attempt to create a measure of diversification across the hedge fund space. Once we have this constructed portfolio of funds, we assume the PM invests equally in all funds for a given month. We calculate the portfolio monthly returns as the equal-weighted average of the selected funds for a given month. We then simulate this exercise 1,000 times for each N and both the random and strategy selection criteria processes, creating a total of 6,000 monthly return portfolios. We repeat the above exercise for the 386,703 fund-of-funds fund-month observations to create similar portfolios and compare these to the fund-level created portfolios.

A. Alpha of Simulated Portfolios

To analyze the return performance of our constructed portfolios, we first calculate each simulated portfolio's outperformance, or alpha, by regressing its time series of monthly returns on the monthly returns of the *US Stocks* and *US Bonds*. We report the distribution of annualized alphas of the 1,000 random selection simulations over the full sample period, 1994-2023, as well as during each sub-period, 1994-2003, 2004-2013, and 2014-2023, for each level of N, graphically in Figure 8. Over the full sample, our fund-level simulated portfolios have similar mean and median annualized alphas, approximately 1.2% for $N = 5, 10$, and 30. As we increase the number of funds selected, a key difference occurs in the higher moments of the annualized alphas, as the standard deviation decreases from 1.5% with $N = 5$ to 0.6% with $N = 30$.

The annualized alphas of the simulated portfolios of individual hedge funds exhibit a similar declining pattern in the sub-periods as with the hedge fund strategy index returns. With each subsequent period, the distribution of annualized alphas shifts to the left, with average alphas falling from 3.0% in 1994-2003 to -1.4% in 2014-2023 when selecting five funds per month. Increasing the number of funds held per month does not provide a meaningful benefit to the mean alpha level, except for in the most recent sub-period. However, larger fund selection reduces the alphas' variance, which is demonstrated visually by observing the overlap of the sub-period distributions. When only five funds are selected per month, the left tail of the 1994-2003 alpha distribution overlaps substantially with the right tail of the 2014-2023 alpha distribution. Even during the worst sub-period for average hedge fund returns, a PM holding five funds per month may perform better than the worst in the best sub-period for average hedge fund returns. This option at higher performance comes at a cost, as selecting only five funds per month also exposes the PM to a well over 50% chance of having negative alpha, even as low as -6.0%. While limiting the upside potential, selecting more monthly funds gives a PM more confidence to achieve the average alpha over the given sub-period. Limited access to top-performing hedge funds is critical in allocation choice feasibility (Binfarè et al., 2022).

We find similar results with the simulated portfolios created with the strategy selection criteria, reported in Figure 9. Over the full sample period, the average annualized alphas is not reliant on the number of funds selected, equal to 1.8% for $N = 5$ and 1.6% for $N = 30$. Likewise, we find a reduction in the standard deviation of the distribution of alphas as we increase the number of funds held per month. While the strategy criteria selection fund portfolios have lower levels of alphas on average, they show an alpha improvement in the first period, 1994-2003, of around 1%. In addition, they benefit from modestly lower standard deviations than the full sample. Thus, a PM choosing to select funds across a diversified approach in terms of strategies may have lower average returns but, in turn, be less exposed to the lowest levels of alphas.

Next, we replicate our individual fund analysis using hedge fund fund-of-funds, where we create portfolios by randomly selecting $N = 1, 2$, or 5 fund-of-funds to invest in each month and simulate this exercise 1,000 times. We report the distribution of the 1,000 simulated portfolios in Figure 10. Across the full sample period, for each level of N, the average annualized alpha is dramatically lower than levels of alpha in the portfolios created from selecting individual funds and is negative, at approximately -0.1%. Regarding the volatility of these alphas, standard deviations are comparable between the portfolios selecting 1, 2, or 5 fund-of-funds and 5, 10, or 30 individual hedge funds, respectively. If we consider diversification as the reduction in the volatility of expected alpha, investing in a single fund-of-fund provides the same diversification benefit as investing in 5-6 individual hedge funds but with a marked reduction in the absolute level of expected annualized alpha of nearly 2%. The drop in alpha is especially noticeable in the 2004-2013 sub-period distribution for the fund-of-funds portfolios compared to the portfolios selected with individual funds. The distribution of alphas for the fund-of-funds portfolios in the 2004-2013 and 2014-2023 sub-periods are much more overlapped than their counterparts in the individual fund portfolios. During the first sub-period, 1994-2003, the fund-of-funds portfolios under-performed the individual fund portfolios in alpha by approximately 1% annually. The under-performance is worse in the 2004-2013 sub-period, with the fund-of-funds portfolios averaging a negative annualized alpha close to -0.1%, while the individual fund portfolios averaged around 2.0%. The most recent sub-period, 2014-2023, demonstrates even worse performance for fund-of-funds, as this average annualized under-performance falls to -2.3%. Our analysis suggests a constrained investor who is limited to only fund-of-funds, on average, would experience the closest relationship to individual fund selection in the most recent sub-period, 2014-2023.

B. *Simulated Portfolios Impact on Utility*

Ultimately, a PM cares about how hedge fund performances impact utility. We examine the Sharpe Ratio and CRRA utility of the 1,000 portfolio simulations to create portfolios that consist of 50% *US Stocks*, 30% *US Bonds*, and 20% hedge funds, referred to as the 50/30/20 portfolios. For reference, the dashed line marks a portfolio's Sharpe Ratio or CRRA utility consisting of 60% *US Stocks* and 40% *US Bonds* over the sample period. We present a summary of the distribution for $N = 10$ for fund selection and $N = 2$ for fund-of-funds in Table VI. We find the 50/30/20 portfolio is an improvement over the baseline 60/40 portfolio in Sharpe Ratio and utility, on average, when including individual funds, regardless of the selection process. However, We find that selection based on strategy yields marginally better than a random draw. Notably, incorporating hedge funds using the strategy selection method results in high levels of Sharpe Ratio for even the 25th percentile simulation, which is not the case for random selection. Access to top-performing funds plays a role in the allocation decision. Fund-of-funds may provide access to hedge funds not otherwise available to a PM. However, while we find a slight increase in utility in the CRRA intertemporal case, allocating to hedge funds via fund-of-funds may hinder the Sharpe Ratio and CRRA terminal utility on average.

In Figure 11 we show the Sharpe Ratio and CRRA intertemporal utility distributions of portfolio simulations for each N of selected funds per month, namely $N = 5, 10$, and 30 for individual funds and, $N = 1, 2$, and 5 for fund-of-funds.

Examining the purely random selection of individual funds, the Sharpe Ratio may be improved by including any amount of randomly selected hedge funds; however, even for $N = 30$ funds, a left-tail event would result in a lower Sharpe Ratio. The same holds for the CRRA utility over the full sample period but with a higher chance of decline in utility. As with the alpha analysis, increasing the number of funds selected per month does not meaningfully change the mean value but reduces the standard deviation of the expected portfolio Sharpe Ratio and CRRA utility. At lower values of N, the PM has greater exposure to the idiosyncratic risks of the selected individual funds. By increasing N, the PM can reduce the noise in the series of monthly returns. When only five funds are selected per month, the PM could expect an average increase to the Sharpe Ratio, but with a possible range of values between 0.4 and 0.6, with values falling below the full sample Sharpe Ratio of the 60/40 stocks/bonds portfolio. In addition, this increase in the return-volatility ratio potentially reduces the portfolio's CRRA utility.

Interestingly, the skewness presents a key difference when comparing the distribution characteristics of the Sharpe Ratio and CRRA utility of the simulated 50/30/20 portfolios. For each value of N, the skewness of the Sharpe Ratio distribution is positive in most sub-periods, while the skewness of the CRRA utility distribution is negative throughout. This suggests the inclusion of hedge funds in any amount exposes the PM to left-tail risk on average, not in the trade-off between returns and volatility, but in the higher moments that impact the CRRA utility of the investor.

Implementing selection criteria for individual hedge funds based on the reporting strategy of the funds yields similar results. With the strategy selection, there is a shift in the distribution of Sharpe Ratios, which implies a strong likelihood of Sharpe Ratio improvement of the portfolio for each value of N. As for the CRRA utility, more of the distribution of the values from the simulated portfolios lies to the right of the 60/40 stocks/bonds portfolio CRRA utility over the full sample, representing a general improvement to utility from the inclusion of hedge funds. However, for $N = 5$ and $N = 10$, a portion of the distribution still falls to the left of this benchmark value. Thus, the PM cannot eliminate the possibility of reducing CRRA utility by including hedge funds based on strategy selection. This slight shift in the distributions is driven in part by the reduction to equity strategy funds, as these constitute around 35% of the full sample of individual hedge funds, which we limit to only 30% in the strategy selection.

Fund-of-funds do not provide the same improvements as individual funds in our setting. Even an investor, including at least five randomly selected fund-of-funds, does not guarantee an increase in the Sharpe Ratio of the 50/30/20 portfolio average. Only selecting 1 or 2 fund-of-funds results in an increased change or reduction in the portfolio Sharpe Ratio. At all values of N, nearly 50% of the CRRA utility distribution for the simulated 50/30/20 portfolios falls below the benchmark value over the full sample period, suggesting an increase in exposure to hedge funds through fund-of-funds may result in the PM experiencing both a reduction in Sharpe Ratio and CRRA utility, as compared to investing in the 60/40 stocks/bonds portfolio.

Figure 12 reports the Sharpe Ratio and CRRA intertemporal utility distributions of the simulated 50/30/20 portfolios over the sub-periods 1994-2003, 2004-2013, and 2014-2023, for fund selection per month of $N = 10$ for the individual hedge funds and $N = 2$ for fund-of-funds. On average, including individual funds in a portfolio by random selection and strategy criteria increases the Sharpe Ratio and CRRA utility in the first two sub-periods, 1994-2003 and 2004-2013. In the most recent sub-period, 2014-2023, the distribution of simulated portfolio Sharpe Ratios is centered well below the benchmark 60/40 stocks/bonds value for that period, while almost all of CRRA Utilities falls to the left of 60/40 stocks/bond benchmark CRRA utility. Only in the first sub-period, 1994-2003, fund-of-funds provide a positive average improvement to the Sharpe Ratio. For the other sub-periods, some portion of the Sharpe Ratio distribution mean and median falls below the 60/40 stocks/bonds value for that period. The CRRA utility exhibits a similar pattern in that for every sub-period, an investor who includes hedge fund exposure through fund-of-funds can at best expect slight gains to utility, but with a near coin-flip chance of declines to utility in 1994-2003 and 2004-2013. This proportion increases, such that in the latest sub-period, 2014-2023, it can almost surely predict a reduction in CRRA utility.

C. Simulated Portfolios with Institutional-Quality Funds

A common critique of hedge fund investing is that average industry performance has deteriorated over time, with alphas declining and risk exposures converging with traditional asset classes. However, this assessment largely reflects the composition of standard commercial databases, which both fail to capture many of the largest, high-quality funds to which institutional investors allocate and incorporate funds that we would not consider investable.[4] As recent work (Barth et al. (2023) and Brown et al. (2025))) has demonstrated, institutional-quality hedge funds - those managing at least $1 billion in assets and with consistent investor access - continue to deliver meaningfully higher alphas, lower systematic risk exposure, and better diversification properties. The exclusion of these funds from typical hedge fund performance analyses means that much of the literature may underestimate the potential benefits of hedge funds in portfolio allocation.

To address this issue, we replicate our previous analysis of hedge fund allocation but now sample exclusively from the set of institutional-quality funds identified in Brown et al. (2025). We follow the same procedure for constructing simulated hedge fund portfolios. We randomly select N individual institutional-quality funds each month, form portfolios based on their realized returns, and conduct 1,000 simulations per selection size. This allows us to compare the risk-return properties of portfolios constructed from these higher-quality funds with those derived from the full hedge fund universe.

We show in Figure 13 the distributions of annualized alphas, Sharpe ratios, and CRRA utility when sampling only from institutional-quality funds in the most recent sub-period, 2014-2023.[5] The top panels highlight a key result: hedge fund alphas differ sharply when focusing solely on institutional-quality funds. The left panel, which includes sampling funds for standard commercial databases, shows a distribution centered around a mean and median alpha of -1.5%, with the lower quartile falling below -2.5% for $N = 10$. Adding more funds from this set reduces dispersion but does little to prevent negative alpha realizations. The right panel, which focuses only (underrepresented) institutional-quality funds, reveals a starkly different outcome - a positive mean and median alpha of approximately 3.5%, with negative alpha occurring only in the lowest 1% of simulations for $N = 5$ and virtually disappearing by $N = 30$. These findings reinforce the critical distinction between commercial databases against the set of funds that institutional investors actually use.

Next, we evaluate the impact on Sharpe ratios when including institutional-quality hedge

[4] Many funds found in commercial databases do not constitute unique primary hedge funds; rather, these include various vehicles such as funds-of-one, managed accounts, redundant share classes, feeder funds, and private asset drawdown funds. Many such entities are either significantly smaller in scale or inactive.

[5] We focus on this period as it aligns with the institutional-quality hedge fund dataset from Brown et al. (2025), constructed to mitigate backfill bias.

funds in a diversified portfolio of 50% *US Stocks*, 30% *US Bonds*, and 20% hedge funds. The middle panels of Figure 13 show the distribution of Sharpe ratios for these 50/30/20 portfolios across different values of N. The left panel, based on hedge funds in commercial databases, demonstrates that hedge fund allocations largely reduce Sharpe ratios relative to the 60/40 stock/bond benchmark (dotted vertical line at 0.538). For $N = 5$, $N = 10$, and $N = 30$, the median Sharpe ratio declines by 5.6%-6.7%, and only the top 5% of $N = 30$ simulations exceed the benchmark. The right panel, which focuses on institutional-quality funds, presents a reversal. Here, a portfolio manager seeking to improve Sharpe ratio in 2014-2023 would have benefited from including institutional-quality hedge funds, with median Sharpe ratios increasing by 12.8%-13.7%. Even the worst-performing $N = 30$ simulation outperformed the 60/40 benchmark, reinforcing the value of high-quality fund selection.

Finally, the bottom panels extend this analysis to CRRA utility. The left panel, based on hedge funds in commercial databases, mirrors the Sharpe ratio results - hedge fund allocations generally reduce utility, with only the top 5% of $N = 10$ simulations seeing any improvement. The right panel, which limits selection to institutional-quality funds, shifts the distribution to the right, suggesting a higher likelihood of utility gains. While not a guaranteed improvement, increasing N from 5 to 30 reduces the probability of underperforming the benchmark from 35% to 15%. As with alphas and Sharpe ratios, the data confirm that institutional-quality hedge funds offer a meaningful advantage, while standard hedge fund databases fail to reflect their potential value.

The key distinction in our findings is that when focusing the investment universe to institutional-quality funds, hedge fund allocations remain beneficial even in recent years when average hedge fund alphas appear to have disappeared. Simulated portfolios drawn from institutional-quality funds exhibit higher Sharpe ratios, lower left-tail risk, and greater stability in alpha compared to those that include the full set of hedge funds. While diversification benefits still decrease as the number of funds shrinks, the deterioration in investor utility is much less pronounced than in the full-universe sample. In contrast to prior results, hedge funds maintain a role in optimal allocations even in the most recent sub-period, provided that the selection process filters out lower-quality funds.

VI. Conclusion

This study explores the optimal allocation of hedge funds within a diversified portfolio of stocks and bonds, emphasizing the strategic role hedge funds play in portfolio construction. Our findings highlight that, even in the absence of an alpha, hedge funds provide meaningful diversification benefits that enhance portfolio efficiency. However, while diversification motives are important, the sensitivity of allocations to alpha remains relevant. Our results demonstrate that small changes in alpha lead to significant shifts in optimal allocations to hedge funds.

Finally, these findings reinforce the critical importance of using the right data when assessing hedge fund allocations. Standard commercial databases systematically omit many institutional-quality hedge funds, leading to an underestimation of hedge fund alpha and an overstatement of their correlation with traditional asset classes. When focusing only on institutional-quality funds, hedge funds retain a meaningful role in optimal portfolio allocation, particularly in improving risk-adjusted returns and preserving diversification benefits. Benchmarking and performance analyses that rely on commercial databases alone may significantly misjudge the potential value of hedge fund allocations.

REFERENCES

Agarwal, Vikas, and Narayan Y. Naik, 2004, Risks and portfolio decisions involving hedge funds, *The Review of Financial Studies* 17, 63–98.

Agarwal, Vikas, Stefan Ruenzi, and Florian Weigert, 2017, Tail risk in hedge funds: A unique view from portfolio holdings, *Journal of Financial Economics* 125, 610–636.

Ang, Andrew, Dimitris Papanikolaou, and Mark M. Westerfield, 2014, Portfolio choice with illiquid assets, *Management Science* 60, 2737–2761.

Asness, Clifford, and Andrea Frazzini, 2013, The devil in hml's details, *Journal of Portfolio Management* 39, 49–68,10.

Asness, Clifford, Andrea Frazzini, and Lasse Heje Pedersen, 2019, Quality minus junk, *Review of Accounting Studies* 24, 34–112.

Bali, Turan G., Stephen J. Brown, and K. Ozgur Demirtas, 2013, Do hedge funds outperform stocks and bonds?, *Management Science* 59, 1887–1903.

Barth, Daniel, Juha Joenväärä, Mikko Kauppila, and Russ Wermers, 2023, A bias-free assessment of the hedge fund industry: A new evaluation of total assets, alphas, and the flow-performance relation, *Working Paper, Board of Governors of the Federal Reserve System* .

Bhardwaj, Geetesh, Gary B. Gorton, and K. Geert Rouwenhorst, 2014, Fooling Some of the People All of the Time: The Inefficient Performance and Persistence of Commodity Trading Advisors, *The Review of Financial Studies* 27, 3099–3132.

Binfarè, Matteo, Gregory Brown, Robert Harris, and Christian Lundblad, 2022, How Does Human Capital Affect Investing? Evidence from University Endowments, *Review of Finance* rfac017.

Bollen, Nicolas P. B., 2013, Zero-r2hedge funds and market neutrality, *Journal of Financial and Quantitative Analysis* 48, 519–547.

Bollen, Nicolas P.B., Juha Joenväärä, and Mikko Kauppila, 2021, Hedge fund performance: End of an era?, *Financial Analysts Journal* 77, 109–132.

Bollen, Nicolas P.B., and Robert E. Whaley, 2009, Hedge fund risk dynamics: Implications for performance appraisal, *The Journal of Finance* 64, 985–1035.

Brandon, Rajna Gibson, and Songtao Wang, 2013, Liquidity risk, return predictability, and hedge funds' performance: An empirical study, *Journal of Financial and Quantitative Analysis* 48, 219–244.

Brown, Gregory W., Philip Howard, and Christian T. Lundblad, 2021a, Crowded Trades and Tail Risk, *The Review of Financial Studies* 35, 3231–3271.

Brown, Gregory W., Wendy Hu, and Bert-Klemens Kuhn, 2021b, Private Investments in Diversified Portfolios, *Working Paper* .

Brown, Gregory W., Christian T. Lundblad, and William Volckmann, 2025, What do we know about institutional-quality hedge funds?, *Working Paper* .

Dichev, Ilia D., and Gwen Yu, 2011, Higher risk, lower returns: What hedge fund investors really earn, *Journal of Financial Economics* 100, 248–263.

Frazzini, Andrea, and Lasse Heje Pedersen, 2014, Betting against beta, *Journal of Financial Economics* 111, 1–25.

Fung, William, and David A. Hsieh, 2001, The risk in hedge fund strategies: Theory and evidence from trend followers, *The Review of Financial Studies* 14, 313–341.

Fung, William, David A. Hsieh, Narayan Y. Naik, and Tarun Ramadorai, 2008, Hedge funds: Performance, risk, and capital formation, *The Journal of Finance* 63, 1777–1803.

Gerakos, Joseph, Juhani T. Linnainmaa, and Adair Morse, 2021, Asset managers: Institutional performance and factor exposures, *The Journal of Finance* 76, 2035–2075.

Getmansky, Mila, Andrew W. Lo, and Igor Makarov, 2004, An econometric model of serial correlation and illiquidity in hedge fund returns, *Journal of Financial Economics* 74, 529–609.

Gilbert, Thomas, and Christopher Hrdlicka, 2015, Why Are University Endowments Large and Risky?, *The Review of Financial Studies* 28, 2643–2686.

Giommetti, Nicola, and Morten Sorensen, 2021, Optimal Allocation to Private Equity, *Working Paper* .

Goetzmann, William N., Elise Gourier, and Ludovic Phalippou, 2019, How alternative are private markets, *Working Paper* .

Gredil, Oleg, Yan Liu, and Berk A. Sensoy, 2021, Diversifying Private Equity, *Working Paper* .

Hochberg, Yael V., and Joshua D. Rauh, 2012, Local Overweighting and Underperformance: Evidence from Limited Partner Private Equity Investments, *The Review of Financial Studies* 26, 403–451.

Jackwerth, Jens Carsten, and Anna Slavutskaya, 2016, The total benefit of alternative assets to pension fund portfolios, *Journal of Financial Markets* 31, 25–42.

Jagannathan, Ravi, Alexey Malakhov, and Dmitry Novikov, 2010, Do hot hands exist among hedge fund managers? an empirical evaluation, *The Journal of Finance* 65, 217–255.

Joenväärä, Juha, Mikko Kauppila, Robert Kosowski, and Pekka Tolonen, 2021, Hedge fund performance: Are stylized facts sensitive to which database one uses?, *Critical Finance Review* 10, 271–327.

Joenväärä, Juha, Robert Kosowski, and Pekka Tolonen, 2019, The effect of investment constraints on hedge fund investor returns, *Journal of Financial and Quantitative Analysis* 54, 1539–1571.

Jorion, Philippe, and Christopher Schwarz, 2019, The fix is in: Properly backing out backfill bias, *The Review of Financial Studies* 32, 5048–5099.

Kosowski, Robert, Narayan Y. Naik, and Melvyn Teo, 2007, Do hedge funds deliver alpha? a bayesian and bootstrap analysis, *Journal of Financial Economics* 84, 229–264.

Kroll, Yoram, Haim Levy, and Harry M. Markowitz, 1984, Mean-variance versus direct utility maximization, *The Journal of Finance* 39, 47–61.

Lerner, Josh, Antoinette Schoar, and Jialan Wang, 2008, Secrets of the academy: The drivers of university endowment success, *The Journal of Economic Perspectives* 22, 207–222.

Pástor, Ľuboš, and Robert F. Stambaugh, 2003, Liquidity risk and expected stock returns, *Journal of Political Economy* 111, 642–685.

Patton, Andrew J., and Tarun Ramadorai, 2013, On the high-frequency dynamics of hedge fund risk exposures, *The Journal of Finance* 68, 597–635.

Sullivan, Rodney N., 2021, Hedge fund alpha: Cycle or sunset?, *The Journal of Alternative Investments* 23, 55–79.

Tables and Figures

Hedge Fund Index Return Summary

	Mean	*St Dev*	*Skew*	*Kurt*	*AR(1)*	*Mean*	*St Dev*	*Skew*	*Kurt*	*AR(1)*
HF Strategy Indices	**Raw Returns**					**Unsmoothed**				
HF All	9.1%	5.9%	-0.75	4.19	0.24	9.0%	6.8%	-0.61	3.61	0.07
HF Credit	8.3%	4.4%	-2.59	17.32	0.37	8.3%	5.4%	-2.47	17.84	0.13
HF Equity	9.6%	7.6%	-0.17	2.46	0.20	9.6%	8.6%	-0.07	2.06	0.06
HF Event Driven	9.8%	6.8%	-1.33	6.48	0.29	9.7%	8.2%	-1.07	6.14	0.09
HF Global Macro	7.3%	5.5%	0.26	1.65	0.10	7.3%	6.1%	0.14	2.05	0.01
HF Managed Futures	7.0%	7.4%	0.33	0.61	0.04	6.9%	7.8%	0.29	0.58	-0.01
HF Multi-strategy	8.3%	4.5%	-1.60	7.46	0.34	8.3%	5.4%	-1.32	6.61	0.13
All Index Average	8.5%	6.0%	-0.83	5.74	0.23	8.4%	6.9%	-0.73	5.56	0.07
Market and Risk Factors										
Global Stocks	8.7%	15.2%	-0.65	1.37	0.04					
Global Bonds	3.9%	5.7%	-0.19	0.95	0.13					
Commodities	6.7%	21.9%	-0.46	2.07	0.15					
Small Stocks (SMB)	0.9%	11.1%	0.64	6.82	-0.05					
Value Stocks (HML)	1.2%	11.7%	0.19	2.36	0.15					
Momentum	4.2%	17.0%	-1.43	9.65	0.04					
Short-term Reversal	3.5%	12.6%	0.33	5.03	-0.10					
Long-term Reversal	1.3%	9.7%	0.56	1.63	0.12					
Illiquidity (Pastor-Stambaugh)	5.1%	13.1%	-0.27	1.27	0.05					
Betting Againt Beta (BAB)	8.2%	13.5%	-0.30	3.23	0.11					
Devil in the Details (DVL)	1.6%	14.0%	0.75	7.24	0.13					
Quality minus Junk (QMJ)	6.0%	9.8%	0.07	1.91	0.20					

Table I. This table presents a summary of monthly nominal returns for Hedge Fund Strategy indices, Market Factors, and Risk Factors. Data for the period January 1994 - June 2023, from various sources as described in Section II. Average returns and standard deviations are annualized. Unsmoothing of returns follows Getmansky, Lo, and Makarov (2004)

Regression Analysis for Hedge Fund Strategy Indices (1994-2023)

	Dep. Var. (Hedge Fund Strategy Index)						
Ind. Var. (Risk Factors)	*All*	*Credit*	*Equity*	*Event Driven*	*Global Macro*	*Managed Futures*	*Multi-Strategy*
Intercept	0.0033***	0.0033***	0.0034***	0.0036***	0.0029***	0.0028**	0.0030***
	(0.0006)	(0.0007)	(0.0007)	(0.0007)	(0.0009)	(0.0011)	(0.0006)
Global Stocks	0.3106***	0.1777***	0.4167***	0.3434***	0.1304***	-0.0046	0.2150***
	(0.0201)	(0.0192)	(0.0223)	(0.0221)	(0.0275)	(0.0387)	(0.0159)
Global Bonds	-0.0237	0.0292	-0.0266	-0.0557	0.0865	0.2450**	0.0327
	(0.0372)	(0.0388)	(0.0413)	(0.0453)	(0.0606)	(0.0961)	(0.0369)
Commodities	0.0379***	0.0235**	0.0347***	0.0290**	0.0260*	0.0553**	0.0318***
	(0.0107)	(0.0103)	(0.0121)	(0.0118)	(0.0142)	(0.0232)	(0.0086)
Small Stocks (SMB)	0.1385***	0.0582***	0.1899***	0.1739***	0.0211	-0.028	0.0957***
	(0.0213)	(0.0209)	(0.0235)	(0.0259)	(0.0327)	(0.0475)	(0.0198)
Value Stocks (HML)	-0.0264	-0.054	-0.021	0.0143	-0.0681	-0.0859	-0.0511
	(0.0406)	(0.0478)	(0.0462)	(0.0529)	(0.0482)	(0.0772)	(0.0403)
Momentum	0.0275	0.0111	0.0369	-0.0113	0.1095***	0.1433***	0.0109
	(0.0234)	(0.0221)	(0.0283)	(0.0271)	(0.0323)	(0.0457)	(0.0189)
Short-term Reversal	-0.0089	0.0248	-0.0293	0.0318	-0.0627**	-0.0994**	0.0019
	(0.0214)	(0.0220)	(0.0257)	(0.0215)	(0.0269)	(0.0455)	(0.0180)
Long-term Reversal	0.015	0.0049	0.0398	0.034	0.0363	0.0133	0.0106
	(0.0301)	(0.0303)	(0.0370)	(0.0354)	(0.0429)	(0.0574)	(0.0246)
Illiquidity (Pastor-Stambaugh)	0.0224	0.0321	0.0324	0.0519**	-0.0071	0.0039	0.0308**
	(0.0174)	(0.0201)	(0.0209)	(0.0211)	(0.0239)	(0.0334)	(0.0152)
Betting Againt Beta (BAB)	0.0760***	0.1113***	0.0472	0.1078***	0.0729***	0.0233	0.1015***
	(0.0239)	(0.0252)	(0.0306)	(0.0286)	(0.0275)	(0.0361)	(0.0244)
Devil in the Details (DVL)	-0.0015	0.0775**	-0.0346	0.0192	0.0707	0.1354*	0.053
	(0.0416)	(0.0394)	(0.0470)	(0.0497)	(0.0541)	(0.0743)	(0.0373)
Quality minus Junk (QMJ)	-0.0872***	-0.1190***	-0.0783**	-0.1234***	-0.0892*	-0.0099	-0.0708***
	(0.0314)	(0.0260)	(0.0365)	(0.0352)	(0.0483)	(0.0630)	(0.0271)
R-squared	0.77	0.65	0.80	0.79	0.26	0.14	0.73
R-squared Adj.	0.76	0.64	0.79	0.78	0.24	0.11	0.72
N	354	354	354	354	354	354	354
IR	0.35	0.35	0.30	0.32	0.19	0.13	0.36

Table II. This table presents regressions of monthly excess returns for Hedge Fund Strategy indices on market factors and risk factors as outlined in Section III. Data for the period January 1994 - June 2023, from various sources as described in Section II. Hedge fund returns unsmoothed following Getmansky, Lo, and Makarov (2004). Standard errors are reported in parentheses. *,**,*** denote statistical significance at the 90%, 95%, and 99% levels respectively.

Hedge Fund Stategy Indices Rolling Alpha and Factor Exposures

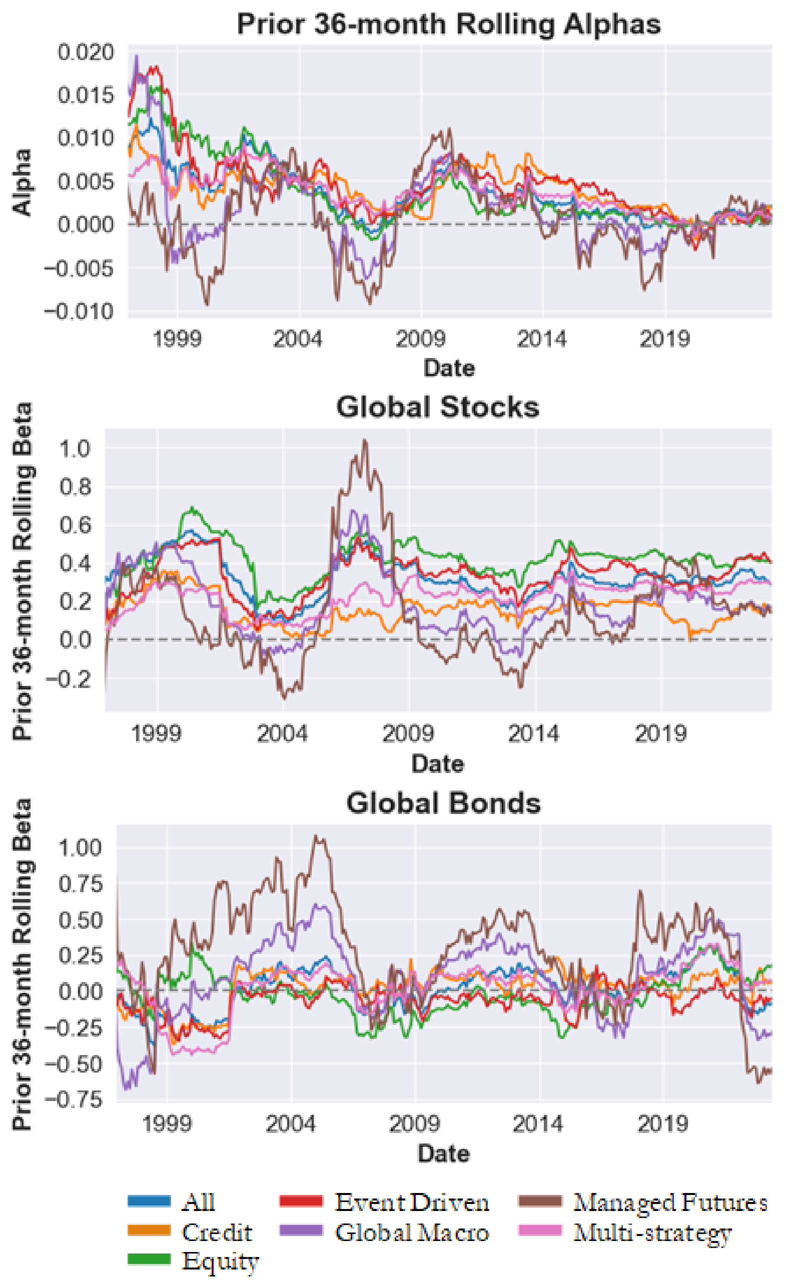

Figure 1. This figure presents rolling alphas and betas for Hedge Fund Strategy indices, estimated from prior 36-month rolling window regressions of monthly excess returns on market factors and risk factors as outlined in Section III. Data for the period January 1994 - June 2023, from various sources as described in Section II . Hedge fund returns unsmoothed following Getmansky, Lo, and Makarov (2004).

Hedge Fund Stragey All Index over Sub-periods

	Dep. Var. (Hedge Fund Strategy All Index)			
	Full Sample	*1994-2003*	*2004-2013*	*2014-2023*
Ind. Var. (Risk Factors)				
Intercept	0.0033***	0.0071***	0.0032***	0.0008
	(0.0006)	(0.0011)	(0.0007)	(0.0006)
Global Stocks	0.3106***	0.3411***	0.2680***	0.3217***
	(0.0201)	(0.0404)	(0.0234)	(0.0223)
Global Bonds	-0.0237	-0.0682	0.0325	-0.0621
	(0.0372)	(0.0695)	(0.0397)	(0.0501)
Commodities	0.0379***	0.0275	0.0229*	0.0212
	(0.0107)	(0.0205)	(0.0138)	(0.0165)
Small Stocks (SMB)	0.1385***	0.1781***	0.1015***	0.0261
	(0.0213)	(0.0387)	(0.0290)	(0.0278)
Value Stocks (HML)	-0.0264	-0.0532	-0.079	0.0042
	(0.0406)	(0.0800)	(0.0588)	(0.0369)
Momentum	0.0275	0.0467	-0.0196	0.0646**
	(0.0234)	(0.0500)	(0.0306)	(0.0274)
Short-term Reversal	-0.0089	-0.0218	-0.0520**	0.0395
	(0.0214)	(0.0276)	(0.0260)	(0.0327)
Long-term Reversal	0.015	-0.0356	-0.0631	-0.0298
	(0.0301)	(0.0661)	(0.0436)	(0.0417)
Illiquidity (Pastor-Stambaugh)	0.0224	-0.0316	0.0024	0.0849***
	(0.0174)	(0.0265)	(0.0224)	(0.0256)
Betting Againt Beta (BAB)	0.0760***	0.0884**	0.0860***	0.1153***
	(0.0239)	(0.0377)	(0.0262)	(0.0291)
Devil in the Details (DVL)	-0.0015	0.0161	-0.0957*	0.1240***
	(0.0416)	(0.0954)	(0.0492)	(0.0465)
Quality minus Junk (QMJ)	-0.0872***	-0.1115**	-0.2622***	-0.0861**
	(0.0314)	(0.0515)	(0.0426)	(0.0417)
R-squared	0.77	0.74	0.88	0.88
R-squared Adj.	0.76	0.71	0.87	0.87
N	354	120	120	114
IR	0.35	0.64	0.46	0.12

Table III. This table presents regressions of monthly excess returns for the Hedge Fund Strategy *All* index on market factors and risk factors as outlined in Section III, across sub-periods. Data for the period January 1994 - June 2023, from various sources as described in Section II . Hedge fund returns unsmoothed following Getmansky, Lo, and Makarov (2004). Standard errors are reported in parentheses. *,**,*** denote statistical significance at the 90%, 95%, and 99% levels respectively.

Hedge Fund Strategy Index Performance Characteristics over Sub-periods

	All	*Credit*	*Equity*	*Event Driven*	*Global Macro*	*Managed Futures*	*Multi-Strategy*
Return (annual)							
Full Sample	6.6%	5.9%	7.2%	7.4%	4.9%	4.6%	5.9%
1994-2003	10.0%	8.2%	12.4%	10.8%	8.7%	7.5%	8.5%
2004-2013	5.8%	6.5%	5.3%	7.4%	3.9%	3.3%	6.1%
2014-2023	4.0%	3.0%	3.8%	3.8%	2.3%	3.1%	3.1%
Volatility (annual)							
Full Sample	6.8%	5.4%	8.5%	8.1%	6.0%	7.7%	5.4%
1994-2003	7.2%	5.0%	9.2%	7.6%	7.8%	9.6%	4.0%
2004-2013	6.6%	5.5%	8.0%	8.0%	5.1%	6.8%	5.9%
2014-2023	6.5%	5.7%	8.1%	8.7%	4.4%	6.3%	6.0%
Sharpe Ratios							
Full Sample	0.95	1.07	0.82	0.88	0.80	0.58	1.07
1994-2003	1.34	1.59	1.28	1.35	1.06	0.75	2.06
2004-2013	0.86	1.15	0.65	0.89	0.74	0.47	1.00
2014-2023	0.60	0.52	0.46	0.43	0.52	0.48	0.51
Alphas (annual)							
Full Sample	4.1%***	4.0%***	4.1%***	4.4%***	3.5%***	3.5%**	3.6%***
1994-2003	8.9%***	7.6%***	11.5%***	9.9%***	6.7%**	5.5%*	8.3%***
2004-2013	3.9%***	4.9%***	3.0%***	5.1%***	3.1%**	3.4%	4.3%***
2014-2023	1.0%	1.1%	0.4%	0.9%	0.3%	0.9%	0.7%
Information Ratios							
Full Sample	0.35	0.35	0.30	0.32	0.19	0.13	0.36
1994-2003	0.64	0.59	0.65	0.62	0.27	0.17	0.83
2004-2013	0.46	0.57	0.33	0.51	0.22	0.16	0.50
2014-2023	0.12	0.14	0.05	0.10	0.03	0.04	0.11
R-squared							
Full Sample	0.77	0.65	0.80	0.79	0.26	0.14	0.73
1994-2003	0.74	0.52	0.75	0.70	0.27	0.23	0.56
2004-2013	0.88	0.82	0.90	0.89	0.45	0.27	0.85
2014-2023	0.88	0.85	0.93	0.91	0.38	0.22	0.91

Table IV. This table presents performance characteristics of monthly excess returns for the Hedge Fund Strategy indices. Returns and standard deviations are annualized. Annualized Alphas, Information Ratios, and R-squareds estimated from regressions on market factors and risk factors as outlined in Section III, across sub-periods. Data for the period January 1994 - June 2023, from various sources as described in Section II . Hedge fund returns unsmoothed following Getmansky, Lo, and Makarov (2004). Standard errors are reported in parentheses. *,**,*** denote statistical significance at the 90%, 95%, and 99% levels respectively.

Hedge Funds added to Heuristic 60/40 Portfolio (stocks/bonds)					
	Hedge Fund Allocation				
	0%	*10%*	*20%*	*30%*	*40%*
Full Sample (1994-2023)					
Return (annual)	4.90%	5.08%	5.25%	5.42%	5.59%
Volatility (annual)	9.67%	9.25%	8.84%	8.46%	8.11%
Sharpe Ratio	0.50	0.54	0.58	0.63	0.67
Skewness	-0.63	-0.64	-0.66	-0.67	-0.69
Kurtosis	1.31	1.40	1.52	1.66	1.85
1994-2003					
Return (annual)	4.30%	4.86%	5.42%	5.99%	6.56%
Volatility (annual)	9.74%	9.33%	8.93%	8.57%	8.24%
Sharpe Ratio	0.43	0.51	0.59	0.68	0.77
Skewness	-0.59	-0.62	-0.65	-0.67	-0.67
Kurtosis	0.02	0.14	0.29	0.48	0.70
2004-2013					
Return (annual)	4.59%	4.71%	4.83%	4.96%	5.08%
Volatility (annual)	9.28%	8.88%	8.51%	8.15%	7.83%
Sharpe Ratio	0.48	0.52	0.56	0.59	0.63
Skewness	-0.91	-0.91	-0.91	-0.90	-0.90
Kurtosis	2.94	2.86	2.76	2.64	2.50
2014-2023					
Return (annual)	5.88%	5.69%	5.50%	5.32%	5.13%
Volatility (annual)	10.07%	9.61%	9.17%	8.74%	8.34%
Sharpe Ratio	0.57	0.58	0.59	0.59	0.60
Skewness	-0.44	-0.45	-0.46	-0.48	-0.51
Kurtosis	1.21	1.40	1.66	2.00	2.44

Table V. This table presents return statistics of diversified portfolios across sample sub-periods consisting of a combination of stocks, bonds, and hedge funds. Columns demonstrate changes in return statistics with larger allocations to hedge funds, represented by the Hedge Fund Strategy *All* index, to the heuristic 60/40 portfolio of stocks and bonds, as outlined in Section IV. Returns are measured in excess of the risk-free rate. Returns and standard deviations are annualized. Data for the period January 1994 - June 2023, from various sources as described in Section II. Hedge fund returns unsmoothed following Getmansky, Lo, and Makarov (2004).

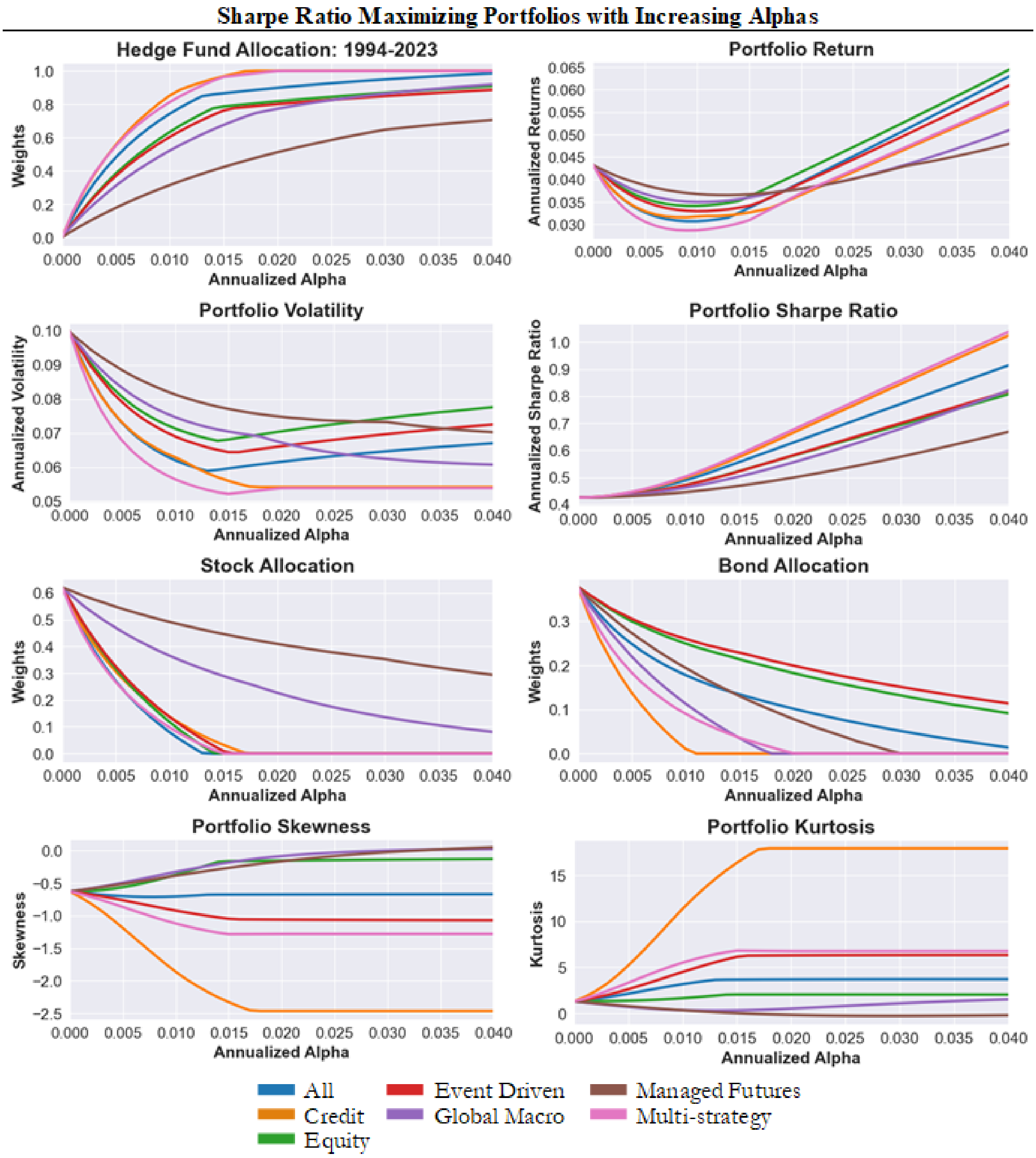

Figure 2. This figure provides portfolio allocations and returns characteristics for Sharpe Ratio optimized portfolios of 3 assets (stocks, bonds, and hedge funds), for increasing levels of hedge fund alpha, as described in Section IV. Returns are measured in excess of the risk-free rate. Data for the period January 1994 - June 2023, from various sources as described in Section II. Hedge fund returns unsmoothed following Getmansky, Lo, and Makarov (2004).

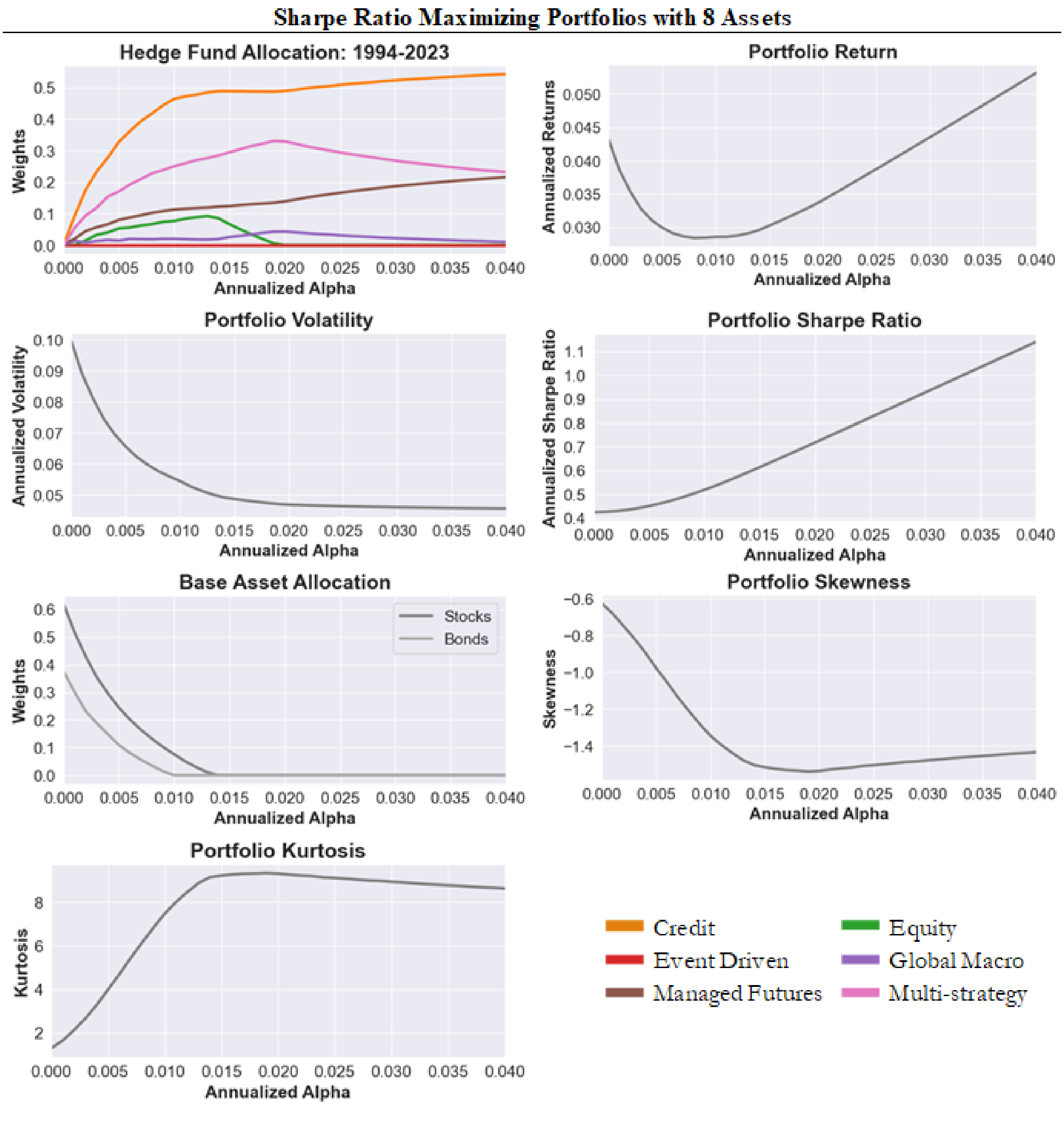

Figure 3. This figure provides portfolio allocations and returns characteristics for Sharpe Ratio optimized portfolios of 8 assets (stocks, bonds, and hedge funds), for increasing levels of hedge fund alpha, as described in Section IV. Returns are measured in excess of the risk-free rate. Data for the period January 1994 - June 2023, from various sources as described in Section II. Hedge fund returns unsmoothed following Getmansky, Lo, and Makarov (2004).

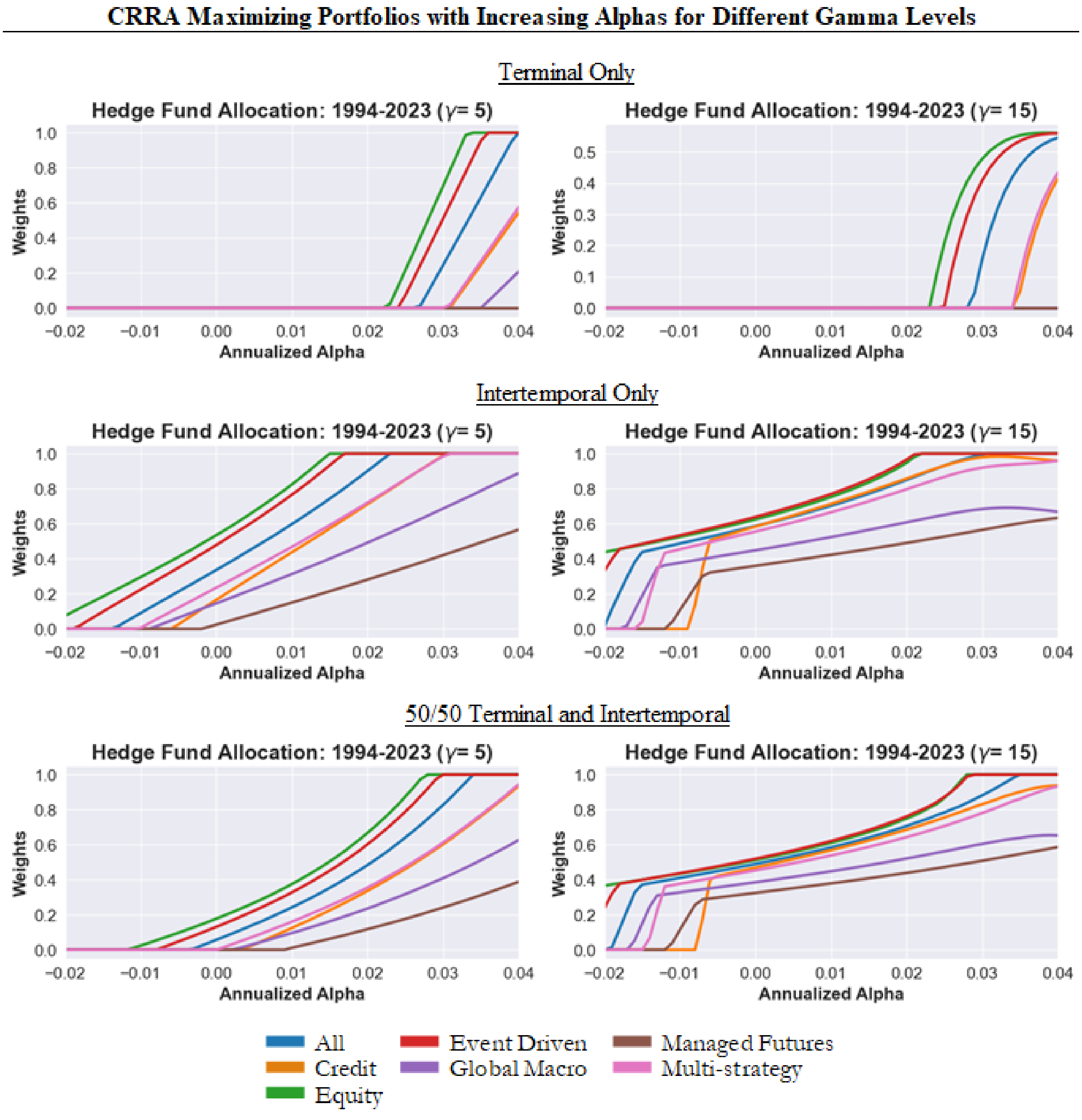

Figure 4. This figure provides a visual representation of hedge fund allocation in optimized portfolios of stocks, bonds, and hedge funds for different levels of risk aversion (γ) and over CRRA Utility preferences, as outlined in Section IV. Returns are measured in excess of the risk-free rate. Data for the period January 1994 - June 2023, from various sources as described in Section II. Hedge fund returns unsmoothed following Getmansky, Lo, and Makarov (2004).

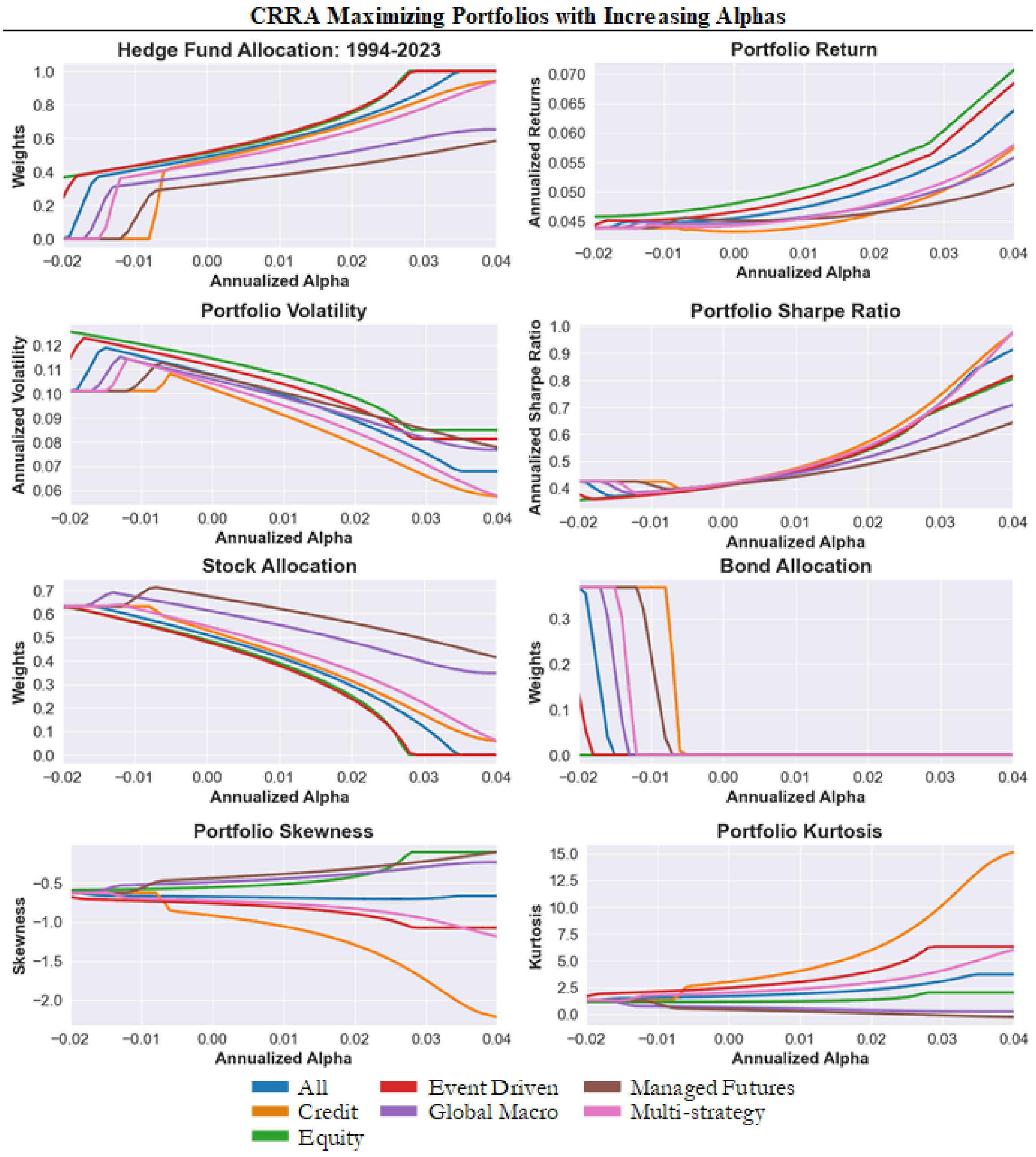

Figure 5. This figure provides a visual representation of portfolio characteristics, including hedge fund allocation, in optimized portfolios of stocks, bonds, and hedge funds for different levels of risk aversion (γ) and over CRRA Utility preferences for both terminal and intertemporal wealth, as outlined in Section IV. Returns are measured in excess of the risk-free rate. Data for the period January 1994 - June 2023, from various sources as described in Section II. Hedge fund returns unsmoothed following Getmansky, Lo, and Makarov (2004).

Sharpe Ratio Maximizing Portfolios over Sub-periods

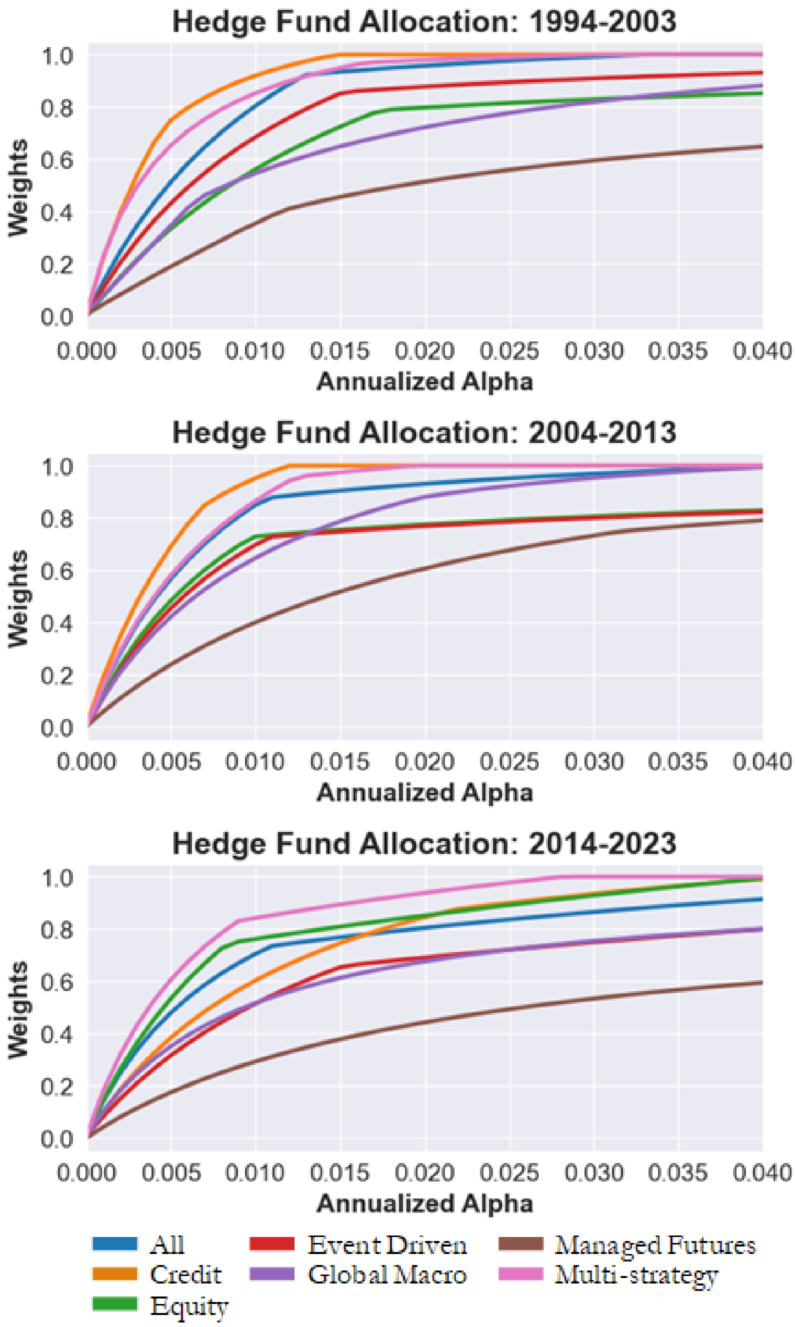

Figure 6. This figure provides a visual representation of hedge fund allocation in Sharpe Ratio optimized portfolios of stocks, bonds, and hedge funds for different sub-periods, as outlined in Section IV. Returns are measured in excess of the risk-free rate. Data for the period January 1994 - June 2023, from various sources as described in Section II. Hedge fund returns unsmoothed following Getmansky, Lo, and Makarov (2004).

CRRA Maximizing Portfolios over Sub-periods

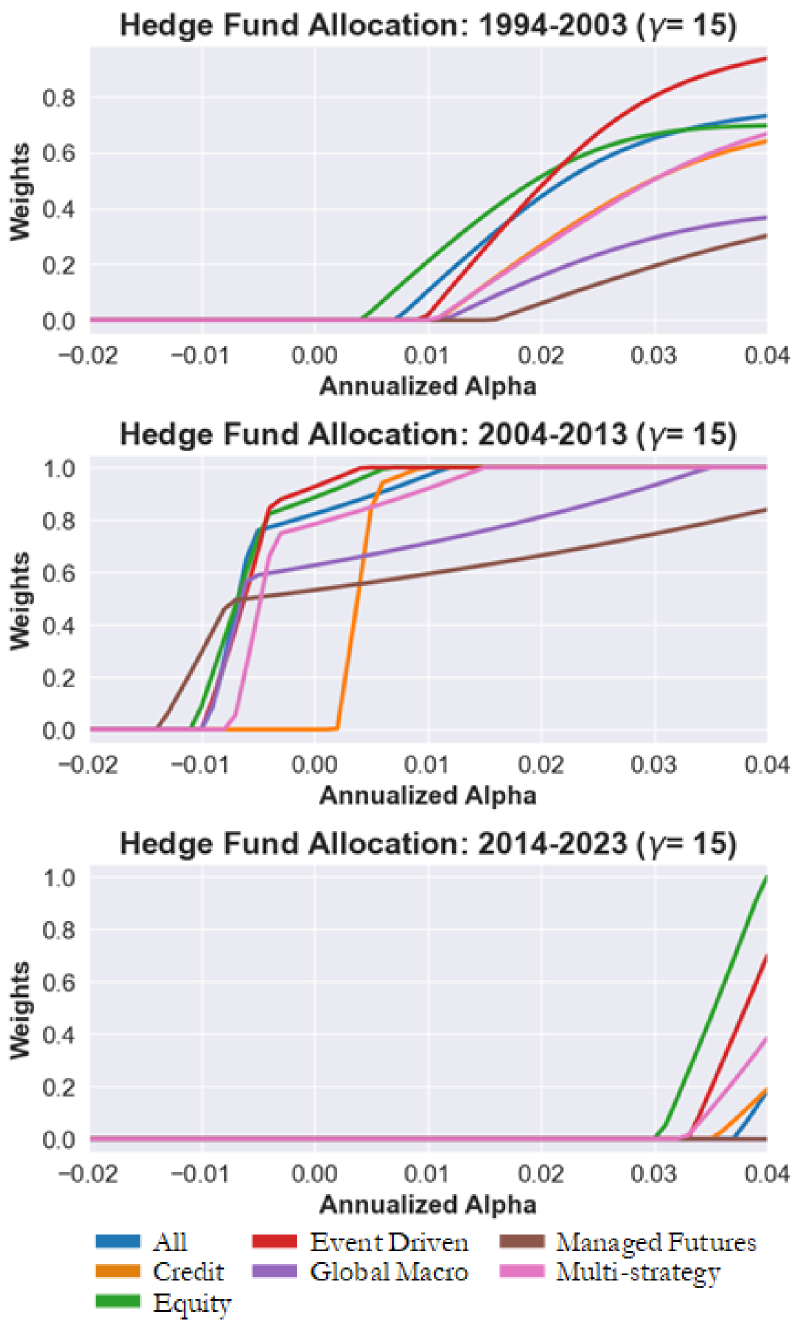

Figure 7. This figure provides a visual representation of hedge fund allocation in optimized portfolios of stocks, bonds, and hedge funds for different sub-periods over CRRA Utility preferences for both terminal and intertemporal wealth, as outlined in Section IV. Returns are measured in excess of the risk-free rate. Data for the period January 1994 - June 2023, from various sources as described in Section II. Hedge fund returns unsmoothed following Getmansky, Lo, and Makarov (2004).

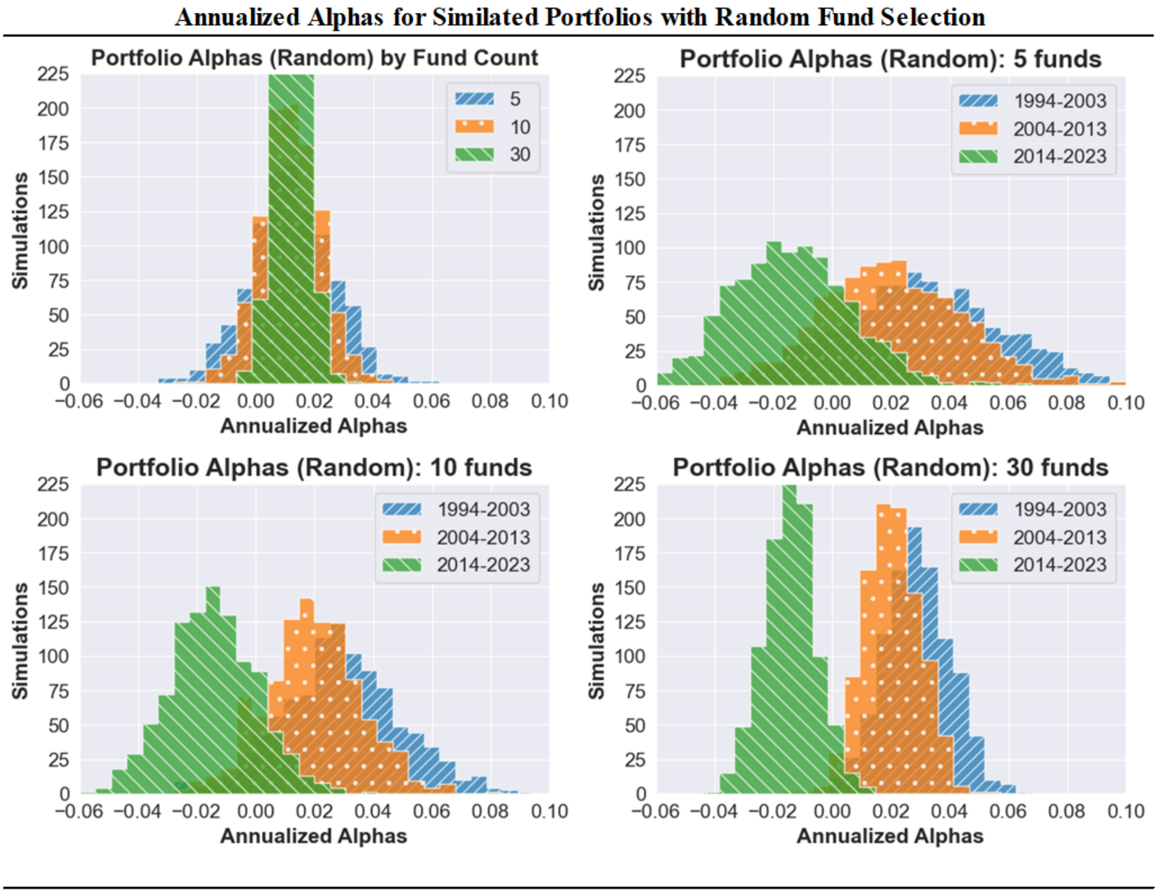

Figure 8. This figure provides distributions of annualized alphas for simulated portfolios of hedge funds of N randomly chosen individual funds, as described in Section V. Returns are measured in excess of the risk-free rate. Data for the period January 1994 - June 2023. Hedge fund returns unsmoothed following Getmansky, Lo, and Makarov (2004).

Annualized Alphas for Similated Portfolios with Strategy Fund Selection

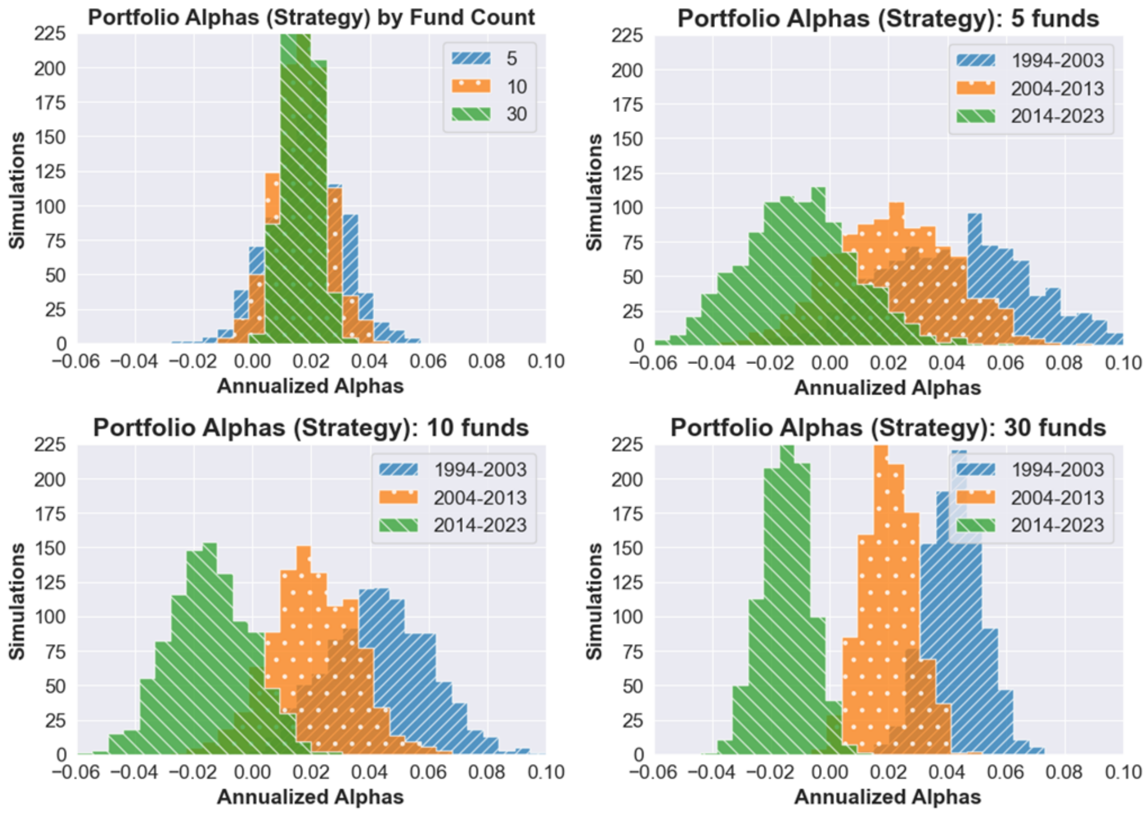

Figure 9. This figure provides distributions of annualized alphas for simulated portfolios of hedge funds of N chosen individual funds using a strategy weight selection criteria, as described in Section V. Returns are measured in excess of the risk-free rate. Data for the period January 1994 - June 2023. Hedge fund returns unsmoothed following Getmansky, Lo, and Makarov (2004).

Annualized Alphas for Similated Portfolios with Random Fund-of-Funds Selection

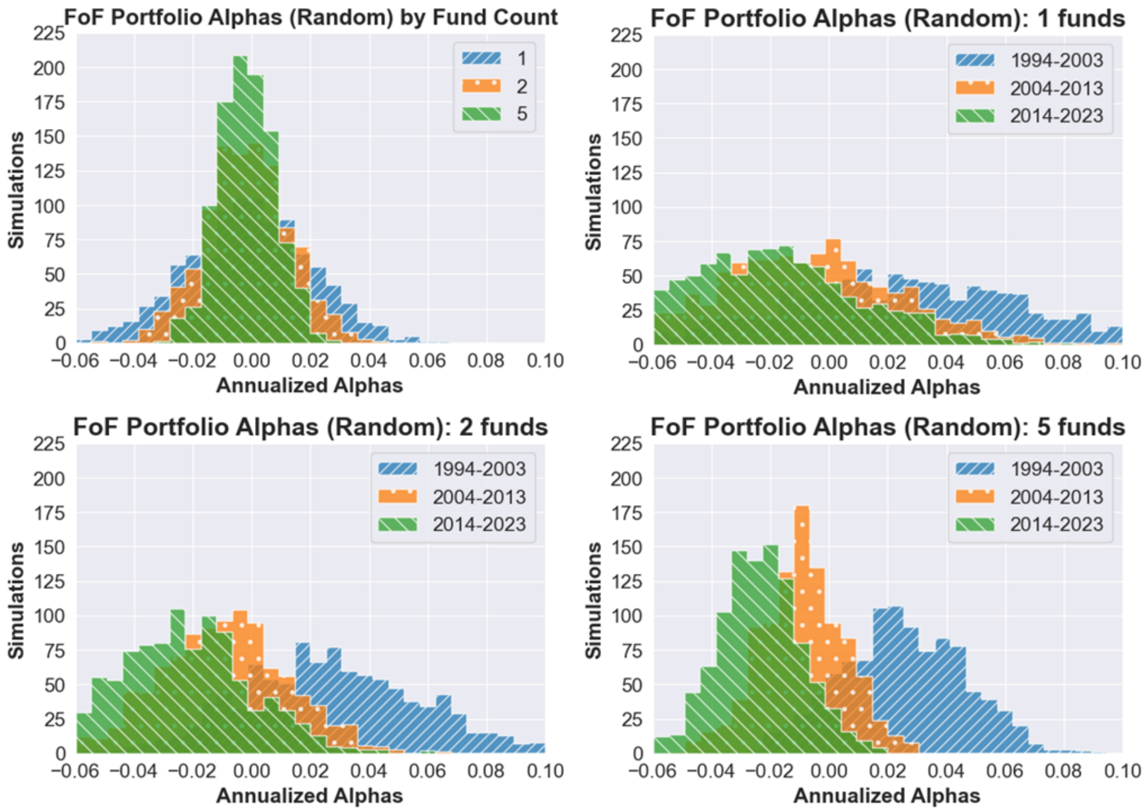

Figure 10. This figure provides distributions of annualized alphas for simulated portfolios of hedge funds of N randomly chosen fund-of-funds, as described in Section V. Returns are measured in excess of the risk-free rate. Data for the period January 1994 - June 2023. Hedge fund returns unsmoothed following Getmansky, Lo, and Makarov (2004).

Simluated Diversified Portfolios Utility Distribution Summary (1994-2023)								
	60/40 Baseline	*Mean*	*St.Dev.*	*p25*	*Median*	*p75*	*Skew*	*Kurt*
Random Selection								
Sharpe Ratio	0.4793	0.4916	0.0215	0.4769	0.4918	0.5062	-0.0444	0.1467
CRRA Terminal	0.2481	0.2479	0.0005	0.2476	0.2480	0.2482	-0.8637	1.7204
CRRA Intertemporal	0.1916	0.1939	0.0051	0.1909	0.1942	0.1974	-0.4765	0.5287
CRRA 50/50 (Both)	0.2198	0.2209	0.0028	0.2193	0.2211	0.2228	-0.5012	0.5850
Strategy Selection								
Sharpe Ratio	0.4793	0.5052	0.0190	0.4925	0.5057	0.5179	-0.0819	0.0749
CRRA Terminal	0.2481	0.2481	0.0004	0.2479	0.2481	0.2484	-0.6955	0.7652
CRRA Intertemporal	0.1916	0.1977	0.0041	0.1950	0.1978	0.2006	-0.1975	-0.0613
CRRA 50/50 (Both)	0.2198	0.2229	0.0022	0.2215	0.2230	0.2245	-0.2238	-0.0388
Fund-of-Funds								
Sharpe Ratio	0.4793	0.4587	0.0308	0.4386	0.4595	0.4794	-0.0298	-0.0043
CRRA Terminal	0.2481	0.2470	0.0010	0.2464	0.2471	0.2477	-1.1245	2.0088
CRRA Intertemporal	0.1916	0.1919	0.0084	0.1867	0.1926	0.1980	-0.5906	0.6725
CRRA 50/50 (Both)	0.2198	0.2195	0.0046	0.2167	0.2198	0.2227	-0.6301	0.7642

Table VI. This table provides a summary of the distributions of Sharpe Ratios and CRRA Utilities for simulated diversified portfolios consisting of stocks, bonds, and hedge funds, for $N = 10$ of individual funds and $N = 2$ fund-of-funds, using either random or strategy weight selection criteria, as outlined in Section V. Returns are measured in excess of the risk-free rate. Data for the period January 1994 - June 2023. Hedge fund returns unsmoothed following Getmansky, Lo, and Makarov (2004).

Sharpe Ratio and CRRA Utility for Simluated Diversifed Portfolios

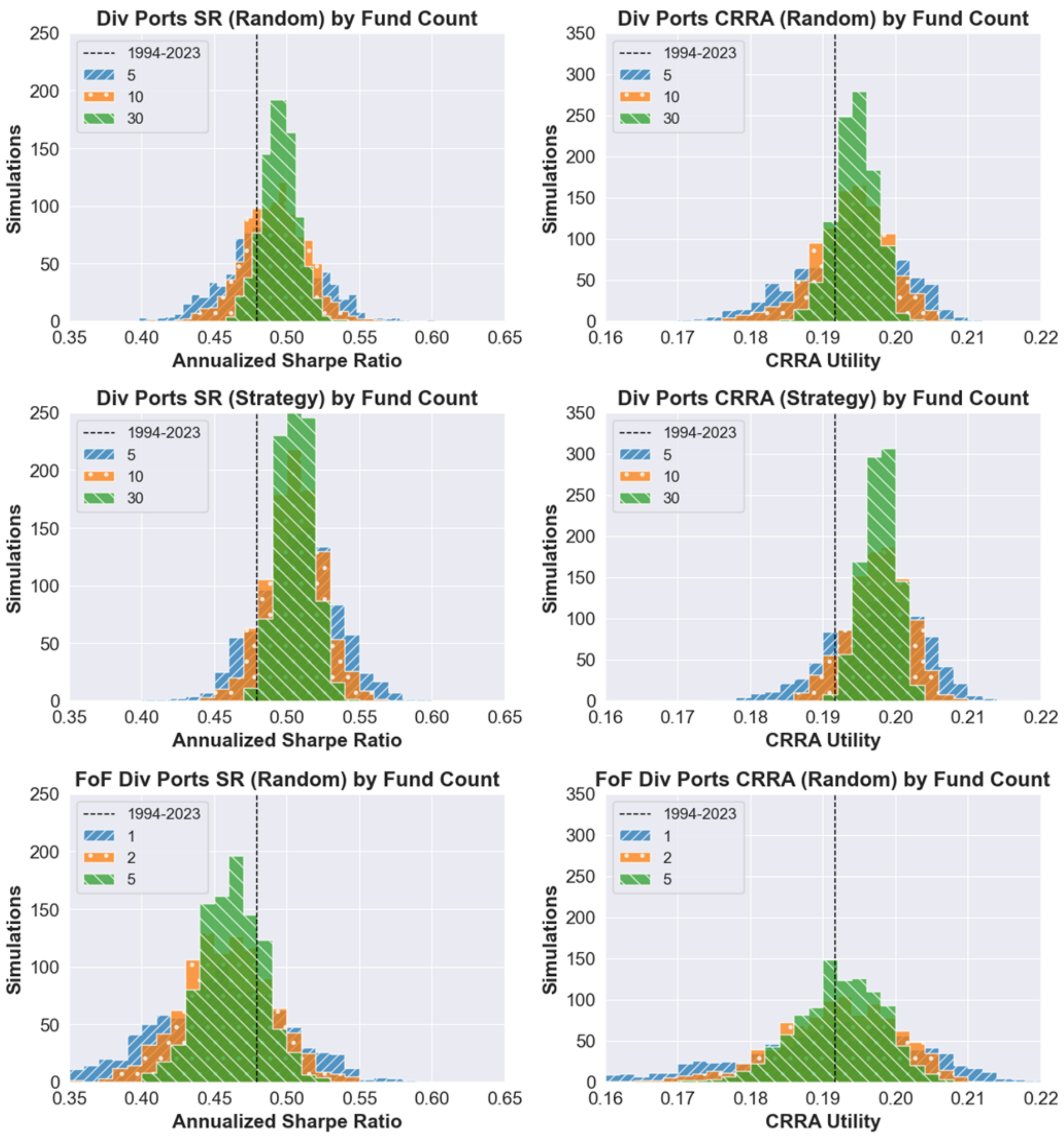

Figure 11. This figure provides a visual representation of the distributions of Sharpe Ratios and CRRA Utilities for simulated diversified portfolios consisting of stocks, bonds, and hedge funds, for different count N of individual funds or fund-of-funds, using either random or strategy weight selection criteria, as outlined in Section V. Returns are measured in excess of the risk-free rate. Data for the period January 1994 - June 2023. Hedge fund returns unsmoothed following Getmansky, Lo, and Makarov (2004).

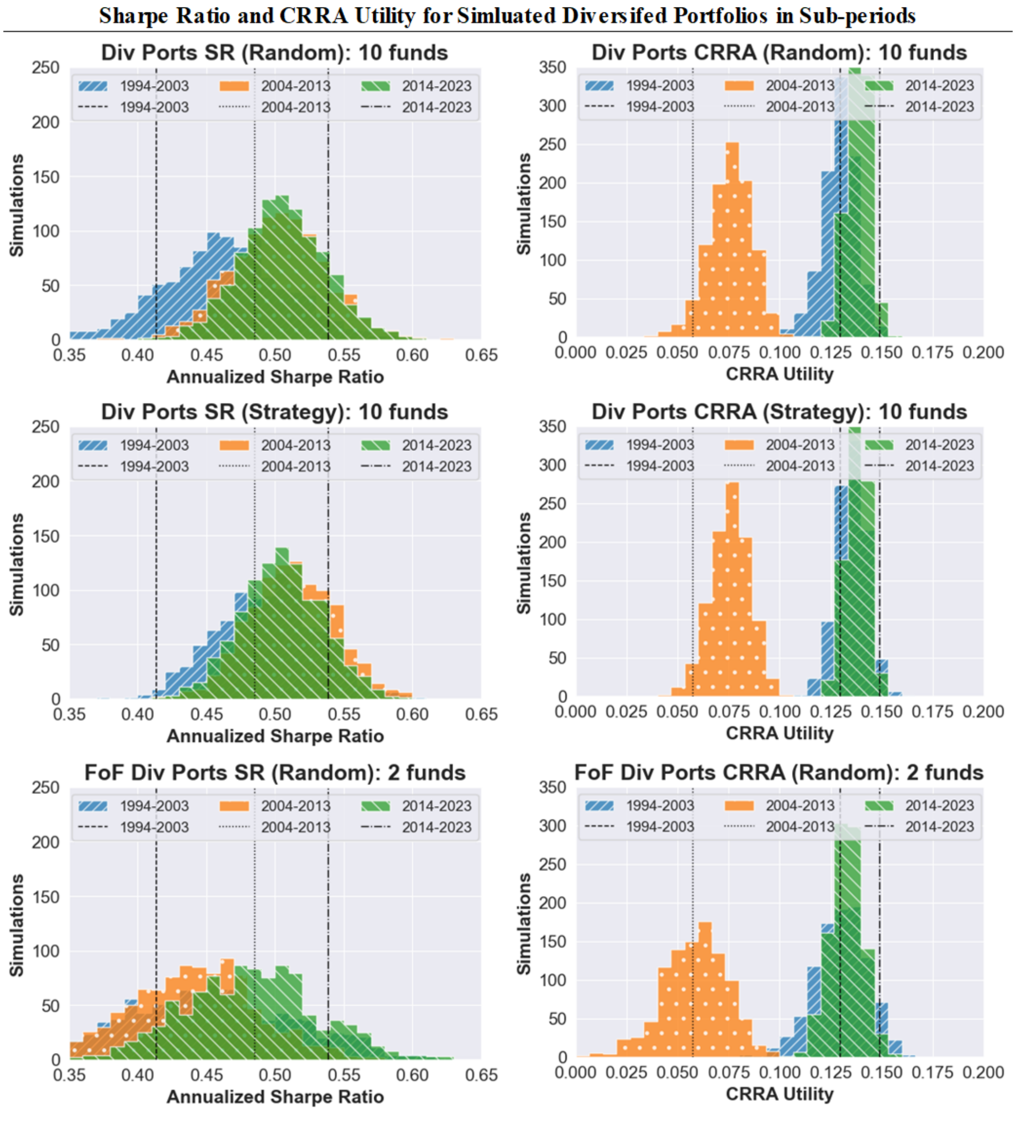

Figure 12. This figure provides a visual representation of the distributions of Sharpe Ratios and CRRA Utilities for simulated diversified portfolios consisting of stocks, bonds, and hedge funds across different sub-periods, for $N = 10$ of individual funds or $N = 2$ fund-of-funds, using either random or strategy weight selection criteria, as outlined in Section V. The vertical lines represent the respective 60/40 stocks/bonds benchmark corresponding to the specified sub-period. Returns are measured in excess of the risk-free rate. Data for the period January 1994 - June 2023. Hedge fund returns unsmoothed following Getmansky, Lo, and Makarov (2004).

All Hedge Funds vs Institutional-Quality Hedge Funds in Simulated Portfolios for 2014-2023

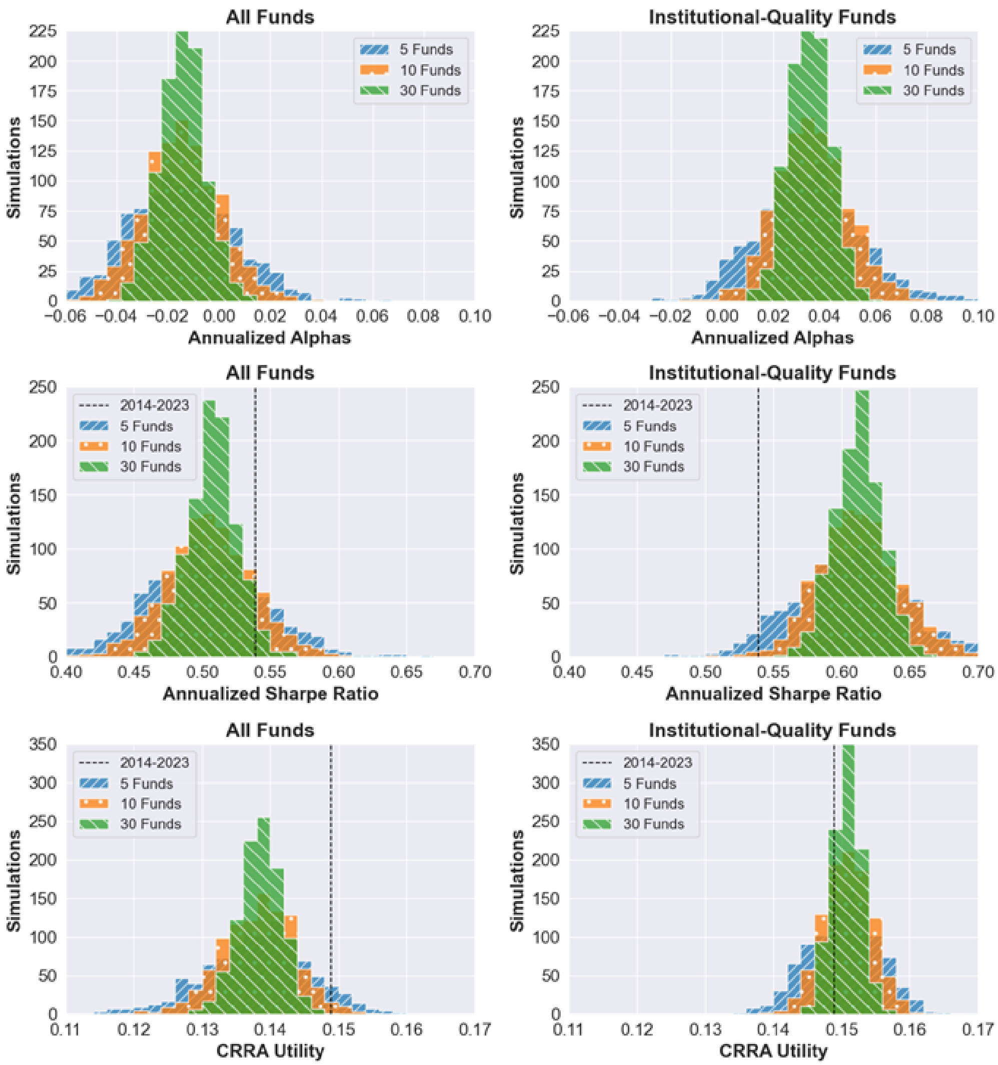

Figure 13. This figure provides a visual comparison between institutional-quality hedge funds and all hedge funds. Representations are given of the distributions of annualized alphas for simulated portfolios of hedge funds as well as annualized Sharpe ratios and intertemporal CRRA utilities for simulated diversified portfolios consisting of 50% stocks, 30% bonds, and 20% hedge funds in the 2014-2023 sub-period, for $N = 5, 10, 30$ of individual funds, using random selection criteria across either all funds or institutional-quality funds, as outlined in Section V. Where present, the vertical lines represent the respective 60/40 stocks/bonds benchmark corresponding to the specified sub-period, 2014-2023. Returns are measured in excess of the risk-free rate. Data for the period January 2014 - June 2023. Hedge fund returns unsmoothed following Getmansky, Lo, and Makarov (2004).